Dynamics of QCD
(Perturbative and Nonperturbative)

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Basics of QCD

More than 90% of our mass is due to QCD: gluons form strings which confine quarks. Quarks (almost massless) with strings create mass of nucleon – 940 MeV. Nucleons in nuclei is source of our weight.

Strings (confinement) is basic element of our world. How strings are formed and how bound states of quarks are arranged – e.g. nucleons or mesons, or bound states of gluons-glueballs?

This is Dynamics of QCD, perturbative (at small distances) and nonperturbative (at large distances, where strings are created).

The QCD Lagrangian looks very simply.

In Euclidean space-time

$$L = \frac{1}{4}(F^a_{\mu\nu})^2 - i\bar{\psi}(\hat{\partial} + m - ig\hat{A})\psi$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$\hat{\partial} = \partial_\mu \gamma_\mu, \quad \hat{A} = A^a_\mu t^a \gamma_\mu$$

The most important requirement: gauge invariance: all physical quantities should be gauge invariant

$$A_\mu(x) = A^a_\mu t^a; \quad F_{\mu\nu}(x) = F^a_{\mu\nu} t^a$$

$$A_\mu(x) \to U^+(x)(A_\mu(x) + \frac{i}{g}\partial_\mu)U(x)$$

$$F_{\mu\nu}(x) \to U^+(x) F_{\mu\nu}(x) U(x).$$

$$\psi(x) \to U^+(x)\psi(x);$$

$$\bar{\psi}(x) \to \bar{\psi}(x) U(x).$$

Examples of gauge invariant quantities;

Local:

$$trF_{\mu\nu}^2(x) = \frac{1}{2}F_{\mu\nu}^a(x)F_{\mu\nu}^a(x), \quad \bar{\psi}(x)\Gamma\psi(x), \quad \bar{\psi}(x)F_{\mu\nu}(x)\psi(x)$$

These create glueball meson hybrid

$$\Gamma = 1, \ \gamma_\mu, \quad \gamma_\mu\gamma_\nu, ...$$

$$\bar{\psi}D_\mu\psi \equiv \bar{\psi}(\partial_\mu - igA_\mu)\psi, \quad D_\mu \to U^+D_\mu U(x)$$

Nonlocal:

first not gauge-invariant, but gauge-covariant parallel transporter

$$\Phi(x,y) = P\exp\int_y^x igA_\mu(z)dz_\mu$$

$$\Phi(x,y) \to U^+(x)\Phi(x,y)U(y)$$

gauge-invariant

$$\bar{\psi}(x)\Phi(x,y)\psi(y),$$

Wilson loop

$$W(C) = trP\exp ig\int_C A_|mu(z)dz_\mu$$

C–closed contour.

Field correlator

$$tr(F_{\mu\nu}(x)\Phi(x,y)F_{\lambda\sigma}(y)\Phi(y,x))$$

One can also define transported quantities from point x to point x_0,

$$G_{\mu\nu}(x,x_0) = \Phi(x_0,x)F_{\mu\nu}(x)\Phi(x,x_))$$

transforming as

$$G_{\mu\nu}(x,x_0) \to U^+(x_0)G_{\mu\nu}(x,x_0)U(x_0).$$

General n-point field correlator

$$D_{\mu_1\nu_1,\dots\mu_n\nu_n}(x_1,\dots x_n) = \langle \mathrm{Tr}\, G_{\mu_1\nu_1}(x_1,x_0)\dots G_{\mu_n\nu_n}(x_n,x_0)\rangle; \tag{1}$$

$$\langle \mathrm{Tr}\, W(C)\rangle = \left\langle \mathrm{Tr}\, \mathcal{P} \exp i \int\limits_S G_{\mu\nu}(z,x_0) d\sigma_{\mu\nu}(z) \right\rangle = \exp \sum_{n=2}^{\infty} i^n \Delta^{(n)}[S], \tag{2}$$

$$\Delta^{(n)}[S] = \frac{1}{n!} \int d\sigma(1)\dots d\sigma(n) D(x_1,\dots x_n)$$

$$\mathcal{G}(x,y) = \langle \phi^\dagger(x)\Phi(x;y)\phi(y)\rangle, \tag{3}$$

$$\mathcal{G}(x,y) = \int\limits_0^\infty ds \int\limits_{z_\mu(0)=x_\mu}^{z_\mu(s)=y_\mu} \mathcal{D}z_\mu \exp\left(-m^2 s - \frac{1}{4}\int\limits_0^s d\tau \left(\frac{dz_\mu(\tau)}{d\tau}\right)^2 \right) \cdot \langle \mathrm{Tr} W(C)\rangle, \tag{4}$$

$$\Delta^{(2)}[S] \gg \sum_{n=3}^{\infty} \Delta^{(n)}[S] \tag{5}$$

$$V(R) = \lim_{T\to\infty} \frac{1}{T}\,\Delta^{(2)}[S = R\times T], \tag{6}$$

Two basic scalar functions D and D_1 define all dynamics with accuracy of $\sim 1\%$

$$D^{(2)}_{\mu\nu\rho\sigma}(z) = 2N_c\left\{(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho})D(z^2)+\right.$$

$$\left.+\frac{1}{2}\left(\frac{\partial}{\partial z_\mu}(z_\rho\delta_{\nu\sigma} - z_\sigma\delta_{\nu\rho}) - \frac{\partial}{\partial z_\nu}(z_\rho\delta_{\mu\sigma} - z_\sigma\delta_{\mu\rho})\right)D_1(z)\right\} \tag{7}$$

$$V_D(R) = 2\int_0^R (R-\lambda)d\lambda\int_0^\infty d\nu D(\lambda,\nu), \tag{8}$$

$$V_1(R) = \int_0^R \lambda d\lambda \int_0^\infty d\nu D_1(\lambda,\nu). \tag{9}$$

$$V_{st}(R) = \sigma R + \mathcal{O}(R^0) \quad ; \quad \sigma = \frac{1}{2}\int d^2 z D(z), \tag{10}$$

D is purely nonperturbative (V.Shevchenko, Yu.S.)

$$D(x) \sim \exp\left(-|x|/\lambda\right). \tag{11}$$

D_1 contains all (but not confinement)

$$D_1 = D_1^{(pert)} + D_1^{(nonpert)}, \tag{12}$$

$$D_1^{(pert)}(x) = \frac{16(\alpha_s + O(\alpha_s^2))}{3\pi x^4} f(x), \quad D_1^{(nonpert)}(x)|_{x\to\infty} \sim \frac{e^{-M_1 x}}{x} \tag{13}$$

$$V_1^{(nonpert)}(R \to \infty) = const = \int_0^\infty \lambda d\lambda \int_0^\infty d\nu D_1(\lambda, \nu), \tag{14}$$

$$V_1^{(nonpert)}(R \ll \lambda) \approx c' R^2; \; V_1^{(pert)}(R \ll \lambda) = -\frac{4(\alpha_s + O(\lambda_s^2))}{3R}. \tag{15}$$

From correlators to Hamiltonian

Path integral for meson Green's function

$$G_{q\bar{q}}(x,y) = \int_0^\infty ds_1 \int_0^\infty ds_2 (Dz)_{xy} (D\bar{z})_{xy} e^{-K_1-K_2} \times$$
$$\times \langle tr \Gamma_{in} (m_1 - \hat{D}_1) W_\sigma (C) \Gamma_{out} (m_2 - \hat{D}_2) \rangle_A, \qquad (16)$$

Wilson loop with spin factors

$$W_\sigma(C) = P_F P_A \exp(ig \int_C A_\mu dz_\mu) \exp(g \int_0^{s_1} \sigma^{(1)}_{\mu\nu} F_{\mu\nu} d\tau_1 - g \int_0^{s_2} \sigma^{(2)}_{\mu\nu} F_{\mu\nu} d\tau_2). \qquad (17)$$

$$\langle W_\sigma(C) \rangle_A \simeq \exp(-\frac{1}{2} [\int_{S_{min}} ds_{\mu\nu}(1) \int_{S_{min}} ds_{\lambda\sigma}(2) +$$
$$+ \sum_{i,j=1}^{2} \int_0^{s_i} \sigma^{(i)}_{\mu\nu} d\tau_i \int_0^{s_j} \sigma^{(j)}_{\lambda\sigma} d\tau_j] \langle F_{\mu\nu}(1) F_{\lambda\sigma}(2) \rangle). \qquad (18)$$

$$\sigma = \frac{1}{2}\int D(x) d^2x. \quad (19)$$

New important variable:(einbein) dynamical mass $\mu_i(t)$

$$2\mu_i(t_i) = \frac{dt_i}{d\tau_i}, \quad \int_0^\infty ds_i (D^4 z^{(i)})_{xy} = const \int D\mu_i(t_i)(D^3 z^{(i)})_{xy}. \quad (20)$$

Last step: from path integral to Hamiltonian

$$G_{q\bar{q}}(x,y) = \langle x|\exp(-HT)|y\rangle \quad (21)$$

$$H_0 = \sum_{i=1}^{2}\left(\frac{m_i^2+\mathbf{p}_i^2}{2\mu_i}+\frac{\mu_i}{2}\right)+\frac{\hat{L}^2/r^2}{2[\mu_1(1-\zeta)^2+\mu_2\zeta^2+\int_0^1 d\beta(\beta-\zeta)^2\nu(\beta)]}+$$

$$+\frac{\sigma^2 r^2}{2}\int_0^1 \frac{d\beta}{\nu(\beta)}+\int_0^1 \frac{\nu(\beta)}{2}d\beta. \quad (22)$$

$$\frac{\partial H_0}{\partial \mu_i}|_{\mu_i=\mu_i^{(0)}} = 0, \quad \frac{\partial H_0}{\partial \nu}|_{\nu=\nu^{(0)}} = 0. \quad (23)$$

$\mu_i^{(0)}$ play role of constituent mass of particle i, $\mu_i^{(0)} = \langle\sqrt{m_i^2 + \mathbf{p}^2}\rangle$

$$H_0(t=0) = \sum_{i=1}^{2} \sqrt{m_i^2 + \mathbf{p}^2} + \sigma r. \tag{24}$$

$$H_0^2 \approx 2\pi\sigma\sqrt{L(L+1)}, \quad \nu^{(0)}(\beta) = \sqrt{\frac{8\sigma L}{\pi}} \frac{1}{\sqrt{1 - 4(\beta - \frac{1}{2})^2}}. \tag{25}$$

$$H_0 \approx H_R + \Delta H_{str}, \quad H_R = \sum_{i=1}^{2} \left(\frac{\mathbf{p}^2 + m_i^2}{2\mu_i} + \frac{\mu_i}{2} \right) + \sigma r \tag{26}$$

String rotation correction

$$\Delta_{str}(L) = \langle \Delta H_{str} \rangle = -\frac{16}{3} \frac{\sigma^2 L(L+1)}{M_0^3} \tag{27}$$

$$\Delta_{str}(L) = -\frac{2\sigma L(L+1)\langle 1/r \rangle}{M_0^2}. \tag{28}$$

Self-energy correction to every quark

$$H_{self} = \sum_{i=1}^{2} \frac{\Delta m_q^2(i)}{2\mu_i}, \quad \Delta m_q^2(i) = -\frac{4\sigma}{\pi}\eta(m_i), \quad \eta(0) \cong 1 \div 0.9, \quad (29)$$

Total Hamiltonian

$$H = H_0 + H_{self} + H_{spin} + H_{Coul} + H_{rad} + H_{mix}. \quad (30)$$

for H_0 only

$$M_0^2 \approx 8\sigma L + 4\pi\sigma(n + \frac{3}{4}), \quad (31)$$

Spin correction H spin

$$\langle W_F \rangle = P_F \exp(\int_0^T \frac{dt}{2\mu(t)} \sigma^{(1)}_{\mu\nu} \frac{\delta}{i\delta s_{\mu\nu}(z(t))})$$

$$\exp(-\int_0^T \frac{d\bar{t}}{z\bar{\mu}(\bar{t})} \sigma^{(2)}_{\mu\nu} \frac{\delta}{i\delta s_{\mu\nu}(\bar{z}(\bar{t}))})\langle W \rangle \tag{32}$$

$$\langle W_F \rangle = P_\sigma \exp(K_1 + K_2 + K_{11} + K_{22} + K_{12})\langle W \rangle \tag{33}$$

$$K_1 = ig^2 \int_0^T \frac{dt}{2\mu(t)} \sigma^{(1)}_{\mu\nu} \int ds^{(w)}_{\lambda\rho} \langle F_{\mu\nu}(z(t)) F_{\lambda\rho}(w) \rangle, \tag{34}$$

$$K_{12} = -g^2 \int_0^T \frac{dt}{2\mu(t)} \int_0^T \frac{d\bar{t}}{2\bar{\mu}(\bar{t})} \sigma^{(1)}_{\mu\nu} \sigma^{(2)}_{\lambda\rho} \langle F_{\mu\nu}(z(t)) F_{\lambda\rho}(\bar{z}(\bar{t})) \rangle \tag{35}$$

$$L_i^{(1)} = (\mathbf{r} \times \mathbf{p}_1)_i = ie_{ikm} r_k(t) \mu \dot{z}_m(t), \tag{36}$$

$$L_i^{(2)} = (\mathbf{r} \times \mathbf{p}_2)_i = ie_{ikm} r_k(t) \bar{\mu} \dot{\bar{z}}_m(t), \tag{37}$$

$$ds_{ik} = (dw_i \dot{w}_k - dw_k \dot{w}_i) = \frac{1}{i} d\beta dt e_{ikm} (\frac{\beta L_m^{(1)}}{\mu} + \frac{(-\beta) L_m^{(2)}}{\bar{\mu}}) \qquad (38)$$

$$V_{SD}(r) = (\frac{\sigma_i^{(1)} L_i^{(1)}}{4\mu^2} - \frac{\sigma_i^{(2)} L_i^{(2)}}{4\bar{\mu}^2})(\frac{1}{r}\frac{d\varepsilon}{dr} + \frac{2}{r}\frac{dV_1}{dr}) +$$

$$\frac{\sigma_i^{(1)} L_i^{(1)} - \sigma_i^{(2)} L_i^{(2)}}{2\mu\bar{\mu}} \frac{1}{r}\frac{dV_2}{dr} + \frac{\sigma_i^{(1)} \sigma_i^{(2)}}{12\mu\bar{\mu}} V_4(r) + \qquad (39)$$

$$\frac{1}{12\mu\bar{\mu}} (3\sigma_i^{(1)} n_i \sigma_k^{(2)} n_k - \sigma_i^{(1)} \sigma_i^{(2)}) V_3(r)$$

All spin potentials V_i expressed through D, D_1

$$\frac{1}{r}\frac{dV_1}{dr} = -\int_{-\infty}^{\infty} d\nu \int_0^r \frac{d\lambda}{r} (1 - \frac{\lambda}{r}) D(\lambda, \nu) \qquad (40)$$

$$\frac{1}{r}\frac{dV_2}{dr} = \int_{-\infty}^{\infty} d\nu \int_0^r \frac{\lambda d\lambda}{r^2} [D(\lambda, \nu) + D_1(\lambda, \nu) + \lambda^2 \frac{\partial D_1}{\partial \lambda^2}] \qquad (41)$$

$$V_3 = -\int_{-\infty}^{\infty} d\nu r^2 \frac{\partial D_1(r,\nu)}{\partial r^2} \tag{42}$$

$$V_4 = \int_{-\infty}^{\infty} d\nu (3D(r,\nu) + 3D_1(r,\nu) + 2r^2 \frac{\partial D_1}{\partial r^2}) \tag{43}$$

$$\frac{1}{r}\frac{d\varepsilon(r)}{dr} = \int_{-\infty}^{\infty} d\nu \int_0^r \frac{d\lambda}{r} [D(\lambda,\nu) + D_1(\lambda,\nu) + (\lambda^2+\nu^2)\frac{\partial D_1}{\partial \nu^2}] \tag{44}$$

$$V_{so}(r) = -\frac{\sigma \mathbf{SL}}{2\mu r} f(r), \quad f(r\to\infty) = 1. \tag{45}$$

$$D(x) = 0, \quad D_1(x) = \frac{16\alpha_s}{3\pi x^4} \tag{46}$$

$$V_{1p} = 0;\ \frac{1}{R}V'_{2p}(r) = \frac{4\alpha_s}{3R^3},\ V_{3p} = \frac{4\alpha_s}{R^3},\ \varepsilon_p(R) = -\frac{4\alpha_s}{3R},\ V_{4p}(R) = \frac{32\pi\alpha_s}{3}\delta^{(3}(\mathbf{R}) \tag{47}$$

$$M_n = \bar{M}_n(\mu_0, \bar{\mu}_0) + \frac{32\mathbf{s}_1\mathbf{s}_2\pi\alpha_s}{9\mu_0\bar{\mu}_0}\varphi_n^2(0) +$$

$$(\frac{\mathbf{s}_1}{2\mu_0^2}+\frac{\mathbf{s}_2}{2\bar{\mu}_0^2})\mathbf{L}\langle\frac{-\sigma f(r)}{r}\rangle+(\frac{\mathbf{S}}{\mu_0\bar{\mu}_0}+\frac{\mathbf{s}_1}{2\bar{\mu}_0^2}+\frac{\mathbf{s}_2}{2\bar{\mu}_0^2})\mathbf{L}\frac{4}{3}\langle\frac{\alpha_s}{r^3}\rangle+$$

$$\frac{\langle 3\mathbf{s}_1\mathbf{n}\mathbf{s}_2\mathbf{n}-\mathbf{s}_1\mathbf{s}_2\rangle}{\mu_0\bar{\mu}_0}\frac{4}{3}\langle\frac{\alpha_s}{r^3}\rangle \tag{48}$$

$$\bar{M}_n(\mu_0,\bar{\mu}_0)=\frac{m^2}{2\mu_0}+\frac{\mu_0}{2}+\frac{\bar{m}^2}{2\bar{\mu}_0}+\frac{\bar{\mu}_0}{2}+\varepsilon(\tilde{\mu}_0)+\Delta_{str}(L)+H_{self}\equiv$$

$$\equiv M_n^{(0)}+\Delta_{str}(L)+H_{self} \tag{49}$$

Baryon $(3q)$ Green's function and Hamiltonian

$$G_{3q}(x^{(i)}|y^{(k)}) = \langle tr_Y \Gamma_{out} \prod_{i=1}^{3} S^{(i)}_{a_i b_i}(x^{(i)}, y^{(i)}) \Gamma_{in} \rangle, \tag{50}$$

$$S(x,y) = (m - \hat{D}) \int_0^\infty ds (Dz)_{xy} e^{-K} W_z(x,y) \exp g \int_0^s \sigma_{\mu\nu} F_{\mu\nu}(z(\tau)) d\tau, \tag{51}$$

$$H^B = H_0^B + H_{self}^B + H_{spin}^B + H_{coul}^B + H_{rad}^B + H_{mix}^B$$

$$H_0^B = \sum_{i=1}^{3} [\frac{m_i^2 + p_{ri}^2}{2\mu_i} + \frac{\mu_i}{2} + \frac{\hat{l}_i^2 / r_i^2}{2(\mu_i + \int_0^1 d\beta_i \beta_i^2 \nu_i(\beta))} +$$

$$+ \frac{\sigma^2}{2} \int_0^1 \frac{d\beta_i}{\nu_i(\beta_i)} \mathbf{r}_i^2 + \frac{1}{2} \int_0^1 \nu_i(\beta_i) d\beta_i]. \tag{52}$$

$$H_0^B = \bar{H}_0 + \Delta H_{string}, \tag{53}$$

$$\bar{H}_0 = \sum_{i=1}^{3} \left(\frac{m_i^2}{2\mu_i} + \frac{\mu_i}{2} \right) + \frac{\mathbf{p}_\xi^2 + \mathbf{p}_\eta^2}{2\mu} + V_{conf}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) \tag{54}$$

$$V_{conf.} = \sigma \sum_{i=1}^{3} |\mathbf{z}^{(i)} - \mathbf{z}^{(Y)}| = \sigma \sum_{i=1}^{3} |\mathbf{z}_i + \mathbf{R} - \mathbf{z}^{(Y)}|, \tag{55}$$

$$\Delta H_{string} = -\sum_{i=1}^{3} \frac{\hat{l}_i^2 \sigma < r_i^{-1} >}{6\mu_i(\mu_i + \frac{1}{3} < \sigma r_i >)} \approx -\sum_{i=1}^{3} \frac{\hat{l}_i^2 \sigma < r_i^{-1} >}{10\mu_i^2}. \tag{56}$$

$$H_{tot} = H_0 + \Delta H_{string} + \Delta H_{coul.} + \Delta H_{self.} + \Delta H_{spin} \tag{57}$$

$$\Delta H_{coul.} = -\frac{2\alpha_s}{3} \sum_{i<j} \frac{1}{|\mathbf{z}^{(i)} - \mathbf{z}^{(j)}|}. \tag{58}$$

$$\Delta H_{self} = -\frac{2\sigma}{\pi} \sum_{i=1}^{3} \frac{\eta_i}{\mu_i}. \tag{59}$$

Baryon lowest states

	$M_{Kn} + \langle \Delta H \rangle_{self}$	$\langle \Delta H \rangle_{coul}$	M^{tot}_{Kn}	M^{tot}(exp)
$K = 0, n = 0$	1.36	-0.274	1.08	1.08
$K = 0, n = 1$	2.19	-0.274	1.91	1.52
$K = 0, n = 2$	2.9	-0.274	262	?
$K = L = 1, n = 0$	1.85	-0.217	1.63	1.6
$K + 2, n = 0$	2.23	-0.186	2.04	?

Radial excitations

$N(939)$ $N(1440)$ $N(1710)$	$\Delta(1232)$ $\Delta(1600)$ $\Delta(1920)$	$\Lambda(1116)$ $\Lambda(1600)$ $\Lambda(1810)$	$\sum(1193)$ $\sum(1660)$ $\sum(1770)$ $\sum(1880)$
$\Delta_1 = 500,$ $\Delta_2 = 770;$	$\Delta_1 = 370,$ $\Delta_2 = 690;$	$\Lambda_1 = 484,$ $\Lambda_2 = 700;$	$\Lambda_1 = 467,$ $\Lambda_2 = 600$

$$M_B^{(i)}(n,L) = \Omega_n^B n + \Omega_L^B L + const, \quad i = N, \Delta, \Lambda, \Sigma \tag{60}$$

Heavy quarkonia

$$\hat{H}_R = \hat{T} + V_0(r), \tag{61}$$

$$\hat{T} = \sqrt{\mathbf{p}^2 + m_1^2} + \sqrt{\mathbf{p}^2 + m_2^2} \tag{62}$$

$$V_0(r) = V_{\rm P}(r) + V_{\rm NP}(r). \tag{63}$$

$$\hat{H}_R \psi(nL) = M_0(nL)\psi(nL), \tag{64}$$

$$M(nL) = M_0(nL) + C_0(m_1, m_2). \tag{65}$$

m_i - current (pole) masses, $C_0(m_1, m_2)$– self-energy corrections

$$m_i = \bar{m}_i(\bar{m}_i), \qquad V_0(r) \equiv V_{\rm NP}(r) \quad (i = 1, 2). \tag{66}$$

$$V_0(r) = V_{\rm P}(n - {\rm loop}) + V_{\rm NP}(r), \tag{67}$$

$$C_0(b\bar{b}) = 0. \tag{68}$$

$$C_0(c\bar{c}) = -30 \pm 10 {\rm MeV} \tag{69}$$

$$V_{\mathrm{NP}}(r) = \sigma_0 \cdot r, \tag{70}$$

$$V_{\mathrm{P}}(r, n - \mathrm{loop}) = -\frac{4}{3}\frac{\alpha_{st}(r, n - \mathrm{loop})}{r} \tag{71}$$

$$\alpha_{st}(r \to \infty) = \alpha_{crit}. \tag{72}$$

$$V_C(r) = \sigma_0 \cdot r - \frac{4}{3}\frac{e}{r}, \tag{73}$$

$$\begin{aligned} \Delta_1 &= \bar{M}(1D) - \bar{M}(1P) \\ \Delta_2 &= \bar{M}(2P) - \bar{M}(1P) \\ \Delta_3 &= \bar{M}(2D) - \bar{M}(2P) \end{aligned} \tag{74}$$

$\Delta_i (i = 1, 2, 3)$	$e_1 = 0.30, n_f = 3$ ( )	$e_2 = 0.4263$	
Δ_1	225	258	262.1±2.2 (exp) $^{+1.0}_{-0.0}$(th)
Δ_2	335	363	360.1±1.2
Δ_3	175	192	

$$\alpha_{st}(r) = \alpha_B(r),$$

Important: Background strong coupling without Landau ghost pole and IR renormalons Yu.S.(1993)

$$\alpha_B(r) = \frac{2}{\pi}\int_0^\infty \frac{dq}{q}\sin(qr)\alpha_B(q) \tag{75}$$

$$\alpha_B(q) = \frac{4\pi}{\beta_0 t_B}\left(1 - \frac{\beta_1}{\beta_0^2}\frac{ln t_B}{t_B}\right). \tag{76}$$

$$t_B = ln\frac{q^2 + M_B^2}{\Lambda_V^2} \tag{77}$$

$$M_B = 1.00 \pm 0.05 \text{ GeV}, \tag{78}$$

$$V_B(r) = \sigma_0 r - \frac{4}{3}\frac{\alpha_B(r)}{r}, \tag{79}$$

$$\Lambda_V(n_f) = \Lambda_{\overline{MS}}(n_f)\exp\left\{\frac{1}{2\beta_0}\left(\frac{31}{3} - \frac{10}{9}n_f\right)\right\} \tag{80}$$

$$\Lambda_{\overline{MS}}^{(5)}(2 - loop) = 240 \pm 25\text{MeV}, \tag{81}$$

$$\Lambda^{(5)}_{\overline{MS}}(3-loop) = 216 \pm 25\mathrm{MeV},$$
$$\Lambda^{(5)}_{V}(2-loop) = 330 \pm 30\mathrm{MeV} \tag{82}$$
$$\Lambda^{(5)}_{V}(2-loop) = 360 \pm 35\mathrm{MeV}$$

$$\alpha_B^{crit}(2-loop) = \alpha_B(q=0) = \frac{4\pi}{\beta_0 t_0}\left(1 - \frac{\beta_1}{\beta_0^2}\frac{lnt_0}{t_0}\right), \tag{83}$$

$$\alpha_B^{crit}(2-loop, n_f = 5) = 0.57 \begin{array}{c} +0.06 \\ -0.05 \end{array}. \tag{84}$$

	$\bar{M}(nL)$	experiment[a)]
1S	3048	3067±1
2S	3667	2673±8
1P	3515	3525±1
$D\bar{D}$		
3S	4075	4040±10
4S	4400	4415±6
2P	3940	3940±12[b)]
1D	3800	$3770(1^3D_1)$
2D	4165	4159±20
1F	4045	–

	$\bar{M}(nL)$	experiment
1S	9466	9460.3±0.3 (1^3S_1)
2S	10023	10023.3±0.3(2^3S_1)
3S	10368	10355.2±0.5(3^3S_1)
1P	9900	9900.1±0.6
2P	10265	10260.0±0.4
1D	10158	10161.2±1.6 (exp) $\begin{matrix}+1.0\\-0.0\end{matrix}$ (th)
2D	10450	–
$B\bar{B}$		
4S	10650	10580.0±3.5
5S	10890	10865±8
6S	11100	11019±8

J^{PC}	M (GeV)	Glueball masses (Kaidalov+Yu.S)	
		Lattice	Lattice
0^{++}	1.58	1.73±0.13	1.74±0.05
0^{++*}	2.71	2.67±0.31	3.14±0.10
2^{++}	2.59	2.40±0.15	2.47±0.08
2^{++*}	3.73	3.29±0.16	3.21±0.35
0^{-+}	2.56	2.59±0.17	2.37±0.27
0^{-+*}	3.77	3.64±0.24	
2^{-+}	3.03	3.1±0.18	3.37±0.31
2^{-+*}	4.15	3.89±0.23	
3^{++}	3.58	3.69±0.22	4.3±0.34
1^{--}	3.49	3.85±0.24	
2^{--}	3.71	3.93±0.23	
3^{--}	4.03	4.13±0.29	

Hybrid Hamiltonian $(q + \bar{q} + g)$

$$H_0^{(hyb)} = \frac{m_1^2}{2\mu_1} + \frac{m_2^2}{2\mu_2} + \frac{\mu_1 + \mu_2 + \mu_g}{2} + \frac{\mathbf{p}_\xi^2 + \mathbf{p}_\eta^2}{2\mu} +$$

$$+\sigma \sum_{i=1}^{2} |\mathbf{r}_g - \mathbf{r}_i| + H_{str} + H_{SE} + H_{spin} + H_c. \tag{85}$$

$$H_c = -\frac{3\alpha_s}{2|r_1 - \mathbf{r}_g|} - \frac{3\alpha_s}{2|\mathbf{r}_2 - \mathbf{r}_g|} + \frac{\alpha_s}{6|\mathbf{r}_1 - \mathbf{r}_2|}. \tag{86}$$

$$K = 0, (\pi + \mathbf{g})1^{+-},\ (\boldsymbol{\rho} + \mathbf{g})(2^{++}, 1^{++}, 0^{++}) \tag{87}$$

$$K = 1, (\pi + (\nabla_x \mathbf{g}))1^{--},\ (\boldsymbol{\rho} + (\nabla_x \mathbf{g}))(2^{-+}, 1^{-+}, 0^{-+}) \tag{88}$$

$$M_0(K = 0) \cong 1.42\text{GeV} \tag{89}$$

$$M_0(K = 1) \cong 1.9\text{GeV} \tag{90}$$

$$M_0(K = 2) \cong 2.45\text{GeV}. \tag{91}$$

Hybrids with two static quarks and one gluon

$$M_{hybrid}^{(long)} = \frac{3}{2^{1/3}} \left(\frac{\sigma}{R}\right)^{1/3} \left(n_z + \frac{1}{2}\right)^{2/3} , \quad M_{hybrid}^{(trans)} = \frac{\sqrt{12}}{R}(n_\perp + \Lambda + 1), \tag{92}$$

$$M_{hybrid}(R) = 2\sqrt{3\sigma} + \frac{\sigma R^2}{2}\sqrt{\frac{\sigma}{3}} + O(R^4). \tag{93}$$

Visualizing confinement and perturbative sources

$$D^{(n)}_{\mu_1\nu_1...\mu_n\nu_n}(x_1,...,x_n,x_0) = \langle \mathrm{Tr}\, G_{\mu_1\nu_1}(x_1,x_0)...G_{\mu_n\nu_n}(x_n,x_0)\rangle \quad (94)$$

Properties of QCD vacuum in gauge-invariant approach

$$W(C) = \mathrm{P}\exp ig \oint_C A^a_\mu(z) t^a dz_\mu \quad (95)$$

$$\Phi(x;y) = \mathrm{P}\exp ig \int_x^y A^a_\mu(z) t^a dz_\mu \quad (96)$$

$$\Phi(x;y) \to \Phi^U(x;y) = U^\dagger(x)\Phi(x;y)U(y) \quad (97)$$

$$G_{\mu\nu}(x,x_0) = \Phi(x_0;x)F_{\mu\nu}(x)\Phi(x;x_0) \quad (98)$$

$$D^{(2)}_{\mu\nu\rho\sigma}(x,y,x_0) = \langle \mathrm{Tr}\, G_{\mu\nu}(x,x_0) G_{\rho\sigma}(y,x_0)\rangle \tag{99}$$

$$D^{(3)}_{\mu\nu\rho\sigma\alpha\beta}(x,y,z,x_0) = \langle \mathrm{Tr}\, G_{\mu\nu}(x,x_0) G_{\rho\sigma}(y,x_0) G_{\alpha\beta}(z,x_0)\rangle \tag{100}$$

$$\langle \mathrm{Tr} W(C)\rangle = \left\langle \mathrm{Tr}\, \mathcal{P} \exp ig \int\limits_S G_{\mu\nu}(z,x_0) d\sigma_{\mu\nu}(z)\right\rangle = \exp \sum_{n=2}^{\infty} (ig)^n \Delta^{(n)}[S] \tag{101}$$

The basic element of Nonperturbative QCD – the correlator $D^{(2)}_{\mu\nu\rho\sigma}$.

$$\Delta^{(2)}[S] = \frac{1}{2} \int\limits_S d\sigma_{\mu\nu}(z_1) \int\limits_S d\sigma_{\rho\sigma}(z_2) D^{(2)}_{\mu\nu\rho\sigma}(z_1,z_2,x_0) \tag{102}$$

If one know Wilson loop $\langle TrW(c)\rangle$, one knows Green's functions, and hence all dynamics

$$\mathcal{G}(x,y) = \langle \phi^{\dagger}(x) \Phi(x;y) \phi(y)\rangle \tag{103}$$

$$\mathcal{G}(x,y) = \int_0^\infty ds \int_{z_\mu(0)=x_\mu}^{z_\mu(s)=y_\mu} \mathcal{D}z_\mu \exp\left(-m^2 s - \frac{1}{4}\int_0^s d\tau \left(\frac{dz_\mu(\tau)}{d\tau}\right)^2\right) \cdot \langle \mathrm{Tr} W(C) \rangle \tag{104}$$

Dominance of Gaussian correlator $D^{(2)} \to$ the QCD vacuum is almost Gaussian.

$$\Delta^{(2)}[S] \gg \sum_{n=3}^{\infty} \Delta^{(n)}[S] \tag{105}$$

$$V(R) = \lim_{T\to\infty} \frac{1}{T}\, g^2 \Delta^{(2)}[S = R \times T] \tag{106}$$

$$D^{(2)}_{\mu\nu\rho\sigma}(z) = \left\langle F^a_{\mu\nu}(0) \cdot \mathrm{P}\exp\left(ig\int_0^1 ds\, z_\mu A^b_\mu(sz) f^{abc}\right) \cdot F^c_{\rho\sigma}(z) \right\rangle \tag{107}$$

Two basic scalars: D and D_1.

$$g^2 D^{(2)}_{\mu\nu\rho\sigma}(z) = (\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho})D(z^2) +$$

$$+\frac{1}{2}\left(\frac{\partial}{\partial z_\mu}(z_\rho\delta_{\nu\sigma} - z_\sigma\delta_{\nu\rho}) - \frac{\partial}{\partial z_\nu}(z_\rho\delta_{\mu\sigma} - z_\sigma\delta_{\mu\rho})\right) D_1(z^2) \qquad (108)$$

$$V(R) = \sigma R + \mathcal{O}(R^0) \quad ; \quad \sigma = \frac{1}{2}\int d^2 z D(z^2) \qquad (109)$$

$$D(z^2) \sim \exp\left(-|z|/\lambda\right) \qquad (110)$$

Mechanism of confinement and dual Meissner effect

$$V^{\mathrm{Coul}}(r) = -\frac{C_F \alpha_s}{r}, \tag{111}$$

$$\boldsymbol{\mathcal{E}}^{\mathrm{Coul}} = \boldsymbol{\nabla} V^{\mathrm{Coul}}(r), \tag{112}$$

The gauge-invariant probe of fields in $W(C)$ with a small plaquette.

The field $\mathcal{F}^{J}_{\mu\nu}(x)$ is gauge invariant.

$$\mathcal{F}^{J}_{\mu\nu}(x) = \langle \mathrm{Tr}\, W(C) \rangle^{-1} \langle \mathrm{Tr}\, i g G_{\mu\nu}(x, x_0) W(C) \rangle. \tag{113}$$

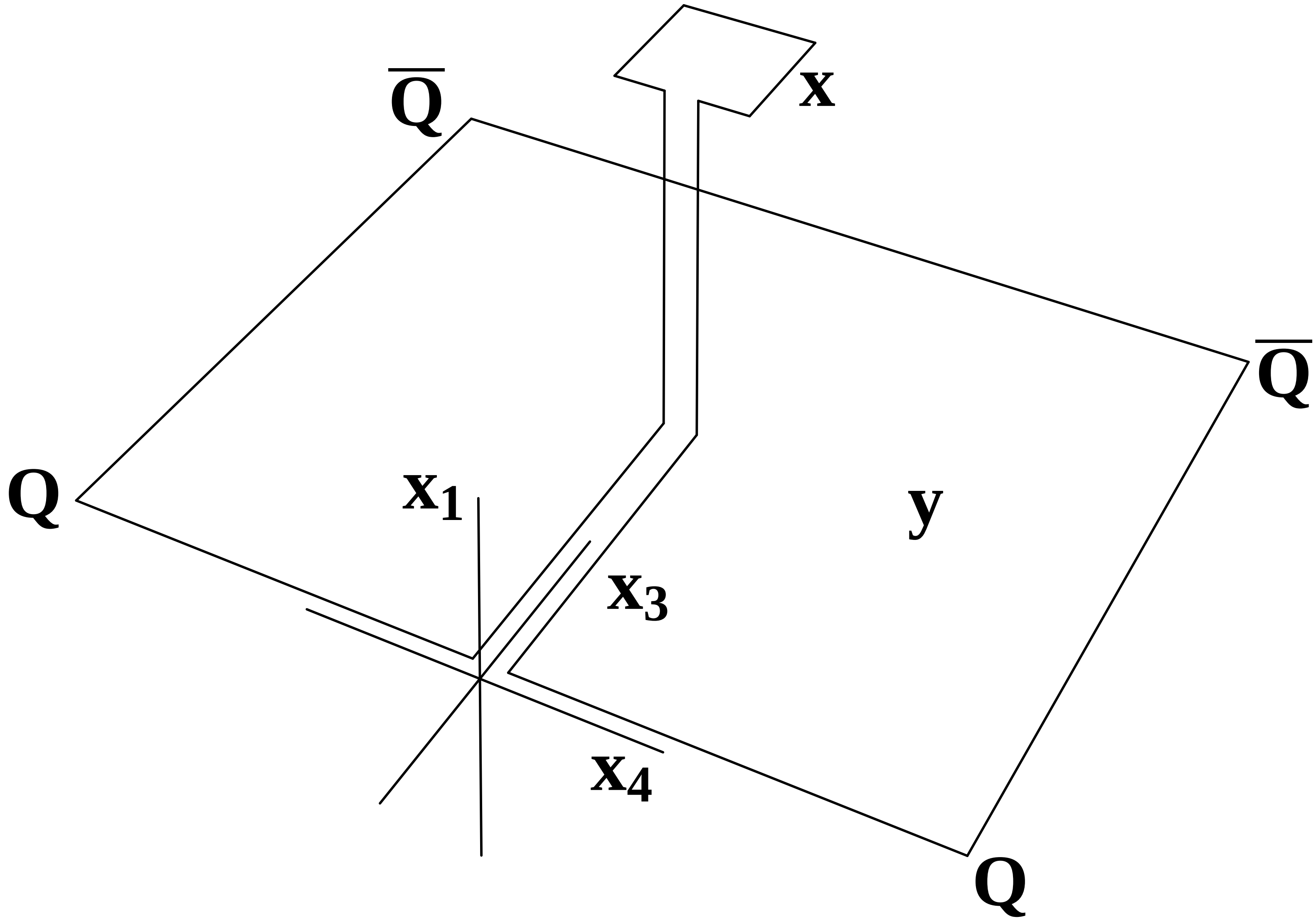

Figure 1: A connected probe (114) for static quark and antiquark

$$W(C,C_P) = W(C_P,x)\Phi(x,x_0)\Phi(x_0,z)W(C,z)\Phi(z,x_0)\Phi(x_0,x) \tag{114}$$

$$\mathcal{F}^J_{\mu\nu}(x)\,\delta\sigma_{\mu\nu}(x) = \langle \mathrm{Tr} W(C)\rangle^{-1}\left(\langle \mathrm{Tr}\, W(C,C_P)\rangle - \langle \mathrm{Tr}\, W(C)\rangle\right) \equiv \tilde{M}(C,C_P) \tag{115}$$

Electric field in the probe $\mathcal{F}^J$; $\varepsilon^J_K \equiv \mathcal{F}^J_{k4}$

$$\mathbf{n}\cdot\boldsymbol{\mathcal{E}}^J(x) = \frac{\tilde{M}(C,C_P)}{a^2}, \tag{116}$$

$g^2 J_\mu(x) = g^2 \int_C dz_\mu \delta^{(4)}(z-x)$. Differentiating $\mathcal{F}^J$ one obtains equations of motion, i.e.Maxwell equations

$$\frac{\partial}{\partial x_\rho}\mathcal{F}^J_{\rho\mu}(x) = g^2 J_\mu(x) \tag{117}$$

$$j^J_\nu(x) = \langle \mathrm{Tr}\, W(C)\rangle^{-1}\Big\{\langle \mathrm{Tr}\,\Phi(x_0;x) ig D_\mu F_{\mu\nu}(x)\Phi(x;x_0)W(C)\rangle$$

$$+g^2 \int_0^1 ds \frac{\partial u_\alpha(s,x)}{\partial s} \frac{\partial u_\beta(s,x)}{\partial x_\mu} \langle \mathrm{Tr}\, [G_{\alpha\beta}(u,x_0) G_{\mu\nu}(x,x_0)] W(C) \rangle \Big\} \tag{118}$$

Magnetic current

$$k_\nu^J(x) = g^2 \langle \mathrm{Tr}\, W(C) \rangle^{-1} \times$$

$$\int_0^1 ds \frac{\partial u_\alpha(s,x)}{\partial s} \frac{\partial u_\beta(s,x)}{\partial x_\mu} \langle \mathrm{Tr}\, [G_{\alpha\beta}(u,x_0) \tilde{G}_{\mu\nu}(x,x_0)]\, W(C) \rangle \tag{119}$$

Maxwell equations with magnetic currents

$$\frac{1}{2}\,\epsilon_{\mu\rho\alpha\beta} \frac{\partial}{\partial x_\rho} \mathcal{F}_{\alpha\beta}^J(x) = k_\mu^J(x) \;\; ; \qquad \frac{\partial}{\partial x_\rho} \mathcal{F}_{\rho\mu}^J(x) = j_\mu^J(x), \tag{120}$$

$$igD_\mu F_{\mu\nu}^a = g^2\, J_\nu^a, \tag{121}$$

where $J^a_\mu(x) = J_\mu(x) T^a, \quad J_\mu(x) = \int_C dz_\mu \delta^{(4)}(z-x)$.

$$j^J_\nu(x) = 4\pi C_F \alpha_s \, J_\nu(x), \qquad (122)$$

Now we use Method of Field correlators, and $\mathcal{F}^J_{\mu\nu}$ is expressed through $D^{(2)}$

$$\mathcal{F}_{\mu\nu}(x) = \int_S d\sigma_{\alpha\beta}(y) \, g^2 D^{(2)}_{\alpha\beta\mu\nu}(x-y), \qquad (123)$$

$$\mathcal{F}_{\mu\nu}(x) = \int_S d^2y \, \mathrm{Tr} \, \langle gF_{\mu\nu}(x)\Phi(x,y)\mathbf{n}g\mathbf{E}(y)\Phi(y,x)\rangle \,, \qquad (124)$$

$$\mathcal{E}_i(\mathbf{r},\mathbf{R}) = n_k \int\limits_0^R dl \int\limits_{-\infty}^{\infty} dt \left(\delta_{ik} D(z) + \frac{1}{2} \frac{\partial z_i D_1(z)}{\partial z_k} \right), \qquad (125)$$

In lowest order

$$\boldsymbol{\mathcal{E}}^{D_1,\mathrm{oge}}(\mathbf{r}) = \boldsymbol{\mathcal{E}}^{\mathrm{Coul}}(\mathbf{r}) - \boldsymbol{\mathcal{E}}^{\mathrm{Coul}}(\mathbf{r}-\mathbf{R}), \qquad (126)$$

$$\boldsymbol{\mathcal{E}}^{\text{Coul}}(\mathbf{r}) = \frac{C_F \alpha_s \mathbf{r}}{r^3}. \tag{127}$$

$$D_1^{\text{oge}}(z) = \frac{4 C_F \alpha_s}{\pi z^4}, \tag{128}$$

We know $D(z^2)$ both from lattice and from analytic calculations

$$D(z^2) = \frac{\sigma}{\pi \lambda^2} \exp\left(-\frac{|z|}{\lambda}\right). \tag{129}$$

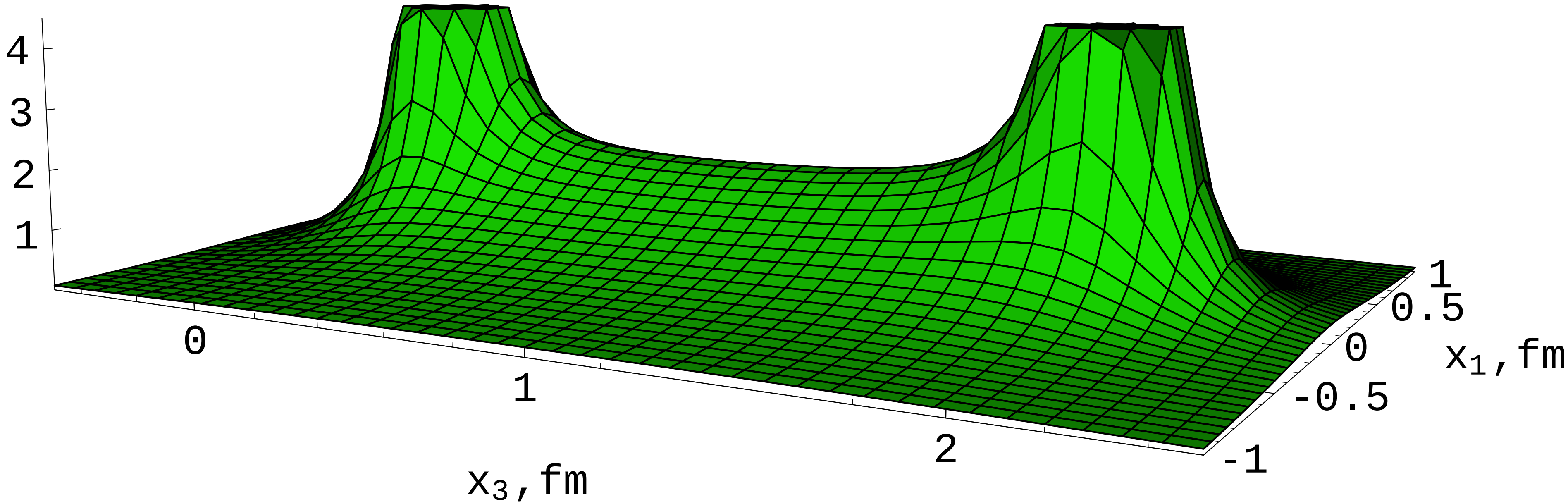

Figure 2: A distribution of the field $|\boldsymbol{\mathcal{E}}(x_1, 0, x_3)|$ (125) at quark-antiquark separation 2 fm. Cutted peaks of color-Coulomb field and string between quark and antiquark are clearly distinguished. The standard values of parameters $\sigma = 0.18$ GeV^2, $\lambda = 0.2$ fm are used.

D_1 contains large perturbative part, and D is purely nonperturbative, therefore one can roughly separate perturbative and nonperturbative in D_1 and D respectively: e.g $\boldsymbol{\mathcal{E}}^D$ and $\boldsymbol{\mathcal{E}}^{D_1}$

$$\boldsymbol{\mathcal{E}}^D(\mathbf{r}, \mathbf{R}) = \mathbf{n} \frac{2\sigma}{\pi} \int\limits_0^{R/\lambda} dl \left| l\mathbf{n} - \frac{\mathbf{r}}{\lambda} \right| K_1 \left(\left| l\mathbf{n} - \frac{\mathbf{r}}{\lambda} \right| \right), \tag{130}$$

$$\boldsymbol{\mathcal{E}}^D(0, \mathbf{R}) = \boldsymbol{\nabla} V^D(R). \tag{131}$$

$$\mathcal{E}(\rho) = 2\sigma \left(1 + \frac{\rho}{\lambda} \right) \exp\left(-\frac{\rho}{\lambda} \right), \tag{132}$$

From Maxwell equations

$$\mathbf{k} = \mathrm{rot}\, \boldsymbol{\mathcal{E}}, \tag{133}$$

We shall see that strings (confinement) are due to $\boldsymbol{\mathcal{E}}^D$, i.e. due to nonperturbative dynamics

$$k_\varphi(\rho) = -\frac{2\sigma\rho}{\lambda^2} \exp\left(-\frac{\rho}{\lambda}\right) . \tag{134}$$

Equivalent of Londons equation

$$\mathrm{rot}\ \mathbf{k} = \lambda^{-2}\boldsymbol{\mathcal{E}} \tag{135}$$

Hence dual Meissner effect: circular magnetic current squeeze (color)electric fluxes.

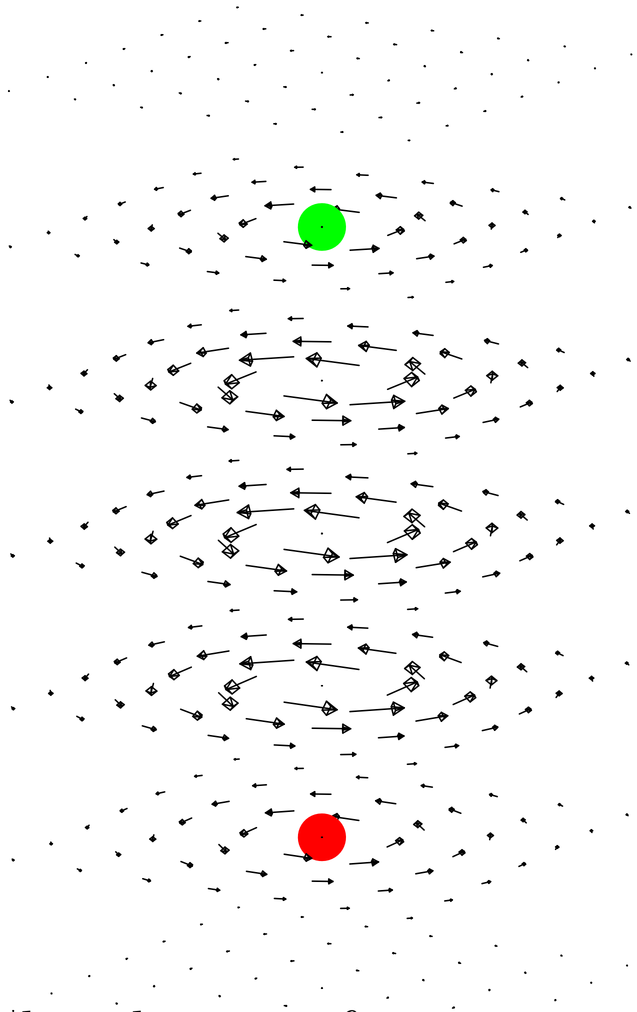

Figure 3: A vector distribution of magnetic currents (130), (133) at quark-antiquark separation 2 fm. Positions of quark and antiquark are shown by points.

$$(\text{rot }\mathbf{k})_z(\rho) = \frac{1}{\rho}\frac{\partial \rho k_\varphi}{\partial \rho} = \gamma(\rho)\,\lambda^{-2}\mathcal{E}(\rho), \tag{136}$$

$$\gamma(\rho) = \frac{-2+\rho/\lambda}{1+\rho/\lambda} \tag{137}$$

$$\text{div}\,\boldsymbol{\mathcal{E}} = \rho, \tag{138}$$

$$\boldsymbol{\mathcal{E}} = \boldsymbol{\mathcal{E}}^{D_1,\text{oge}} + \boldsymbol{\mathcal{E}}^{D_1,\text{np}} + \boldsymbol{\mathcal{E}}^{D}, \tag{139}$$

$$\text{div}\,\boldsymbol{\mathcal{E}}^{D_1,\text{np}} = -\text{div}\,\boldsymbol{\mathcal{E}}^{D}, \tag{140}$$

$$\rho = 4\pi C_F \alpha_s\,(\delta(\mathbf{r}) - \delta(\mathbf{r}-\mathbf{R})). \tag{141}$$

$$\text{div}\,\boldsymbol{\mathcal{E}}^{D_1,\text{np}} = \tilde{\rho}(r) - \tilde{\rho}(|\mathbf{r}-\mathbf{R}|), \tag{142}$$

$$\tilde{\rho}(r) = -\frac{2\sigma}{\pi\lambda^2}\,r\,K_1\left(\frac{r}{\lambda}\right). \tag{143}$$

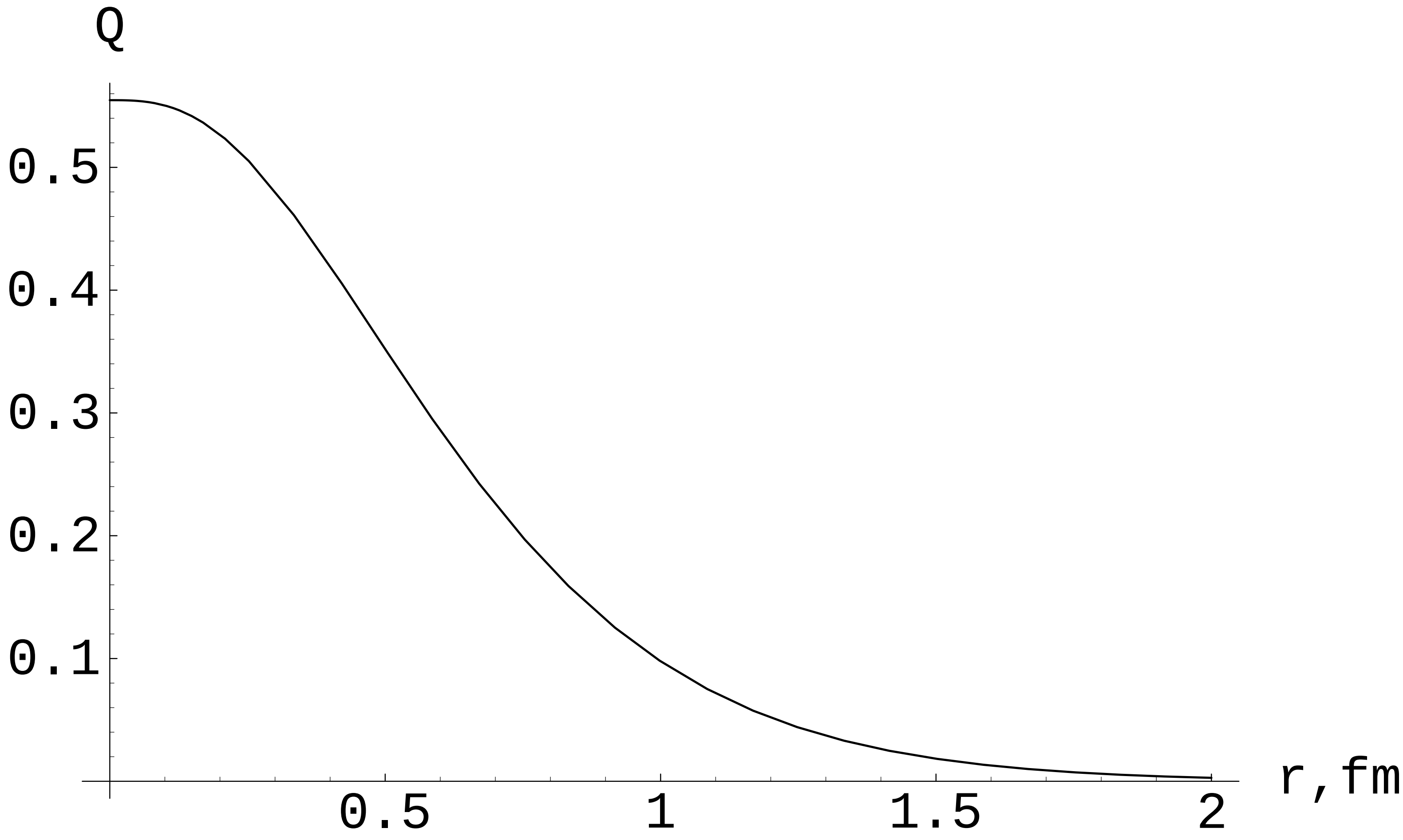

Figure 4: An effective charge $Q(r)$ (146) in dependence of the distance from the quark for $\sigma = 0.18$ GeV^2, $\lambda = 0.2$ fm and constant value $\alpha_s = 0.42$.

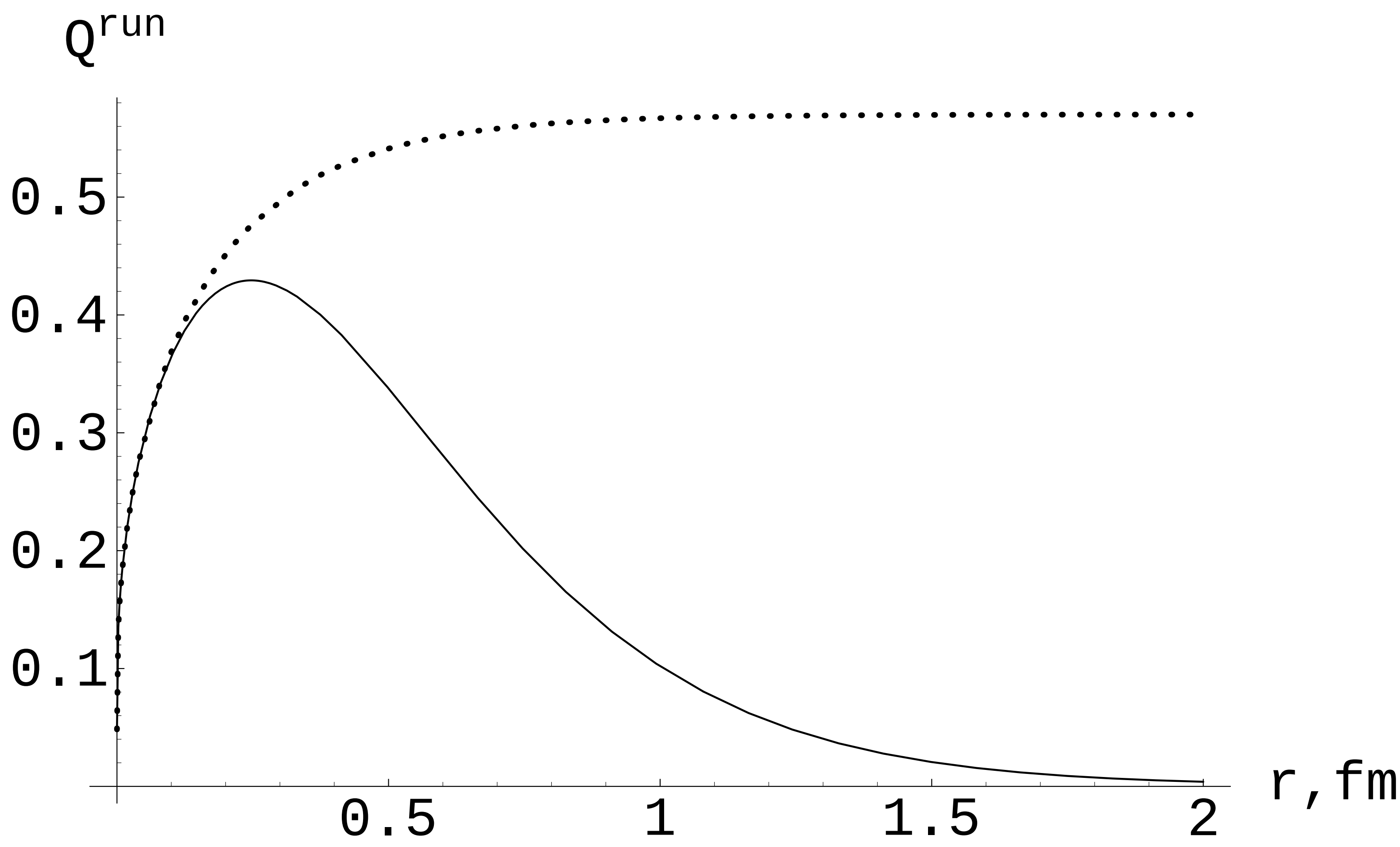

Figure 5: A running background coupling $C_F\alpha_B(r)$ (dotted curve) and running effective charge $Q^{\rm run} = C_F\alpha_B(r) - \tilde{Q}(r)$ (solid curve) vs. the distancefrom the quark.

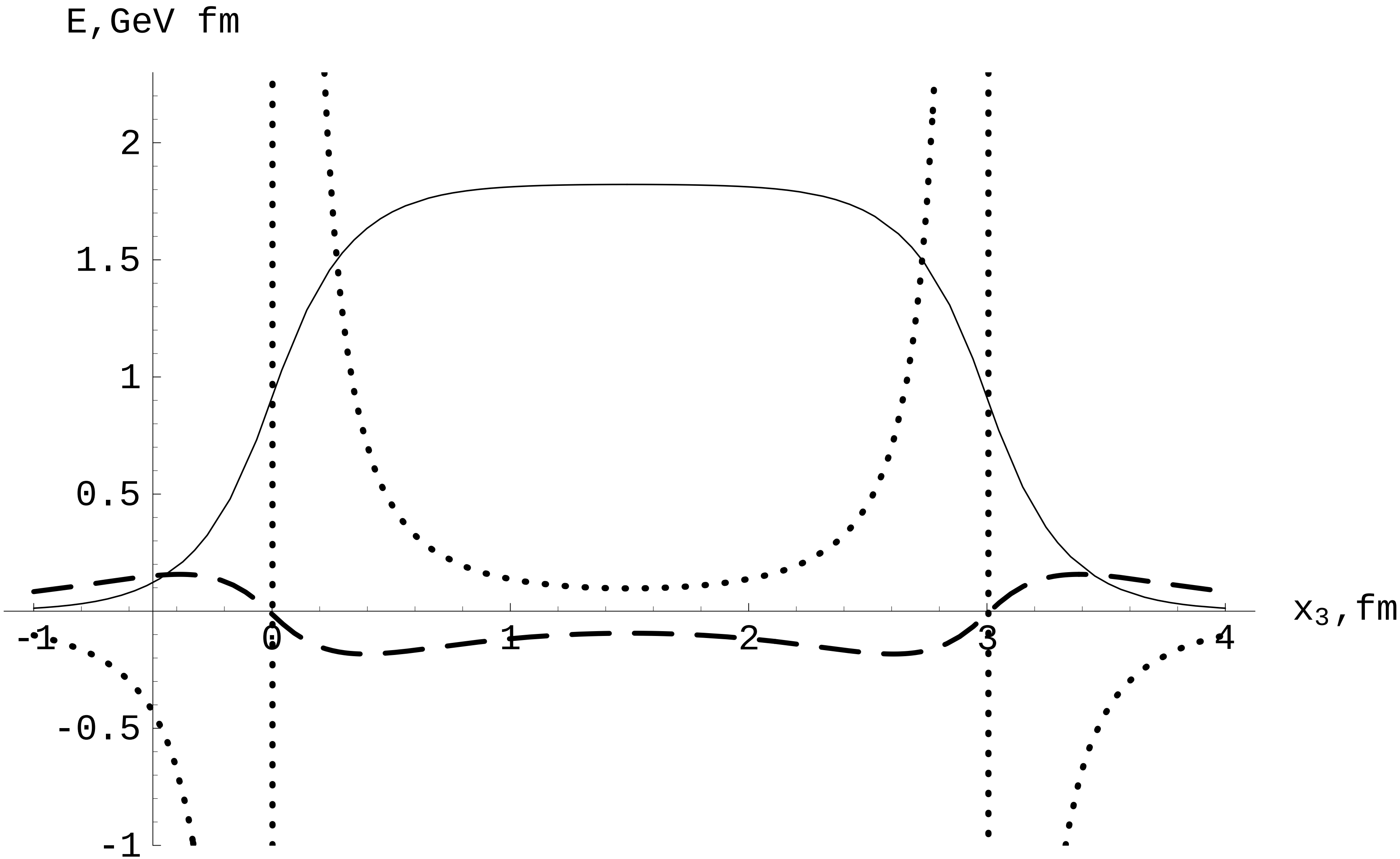

Figure 6: Distributions of the projections of the fields $\boldsymbol{\mathcal{E}}^{D}(0,0,x_3)$ (solid curve), $\boldsymbol{\mathcal{E}}^{D_1,\mathrm{np}}(0,0,x_3)$ (dashed curve) and $\boldsymbol{\mathcal{E}}^{D_1,\mathrm{oge}}(0,0,x_3)$ (dotted curve) onto the quark-antiquark axis at $Q\bar{Q}$-separation 3 fm.

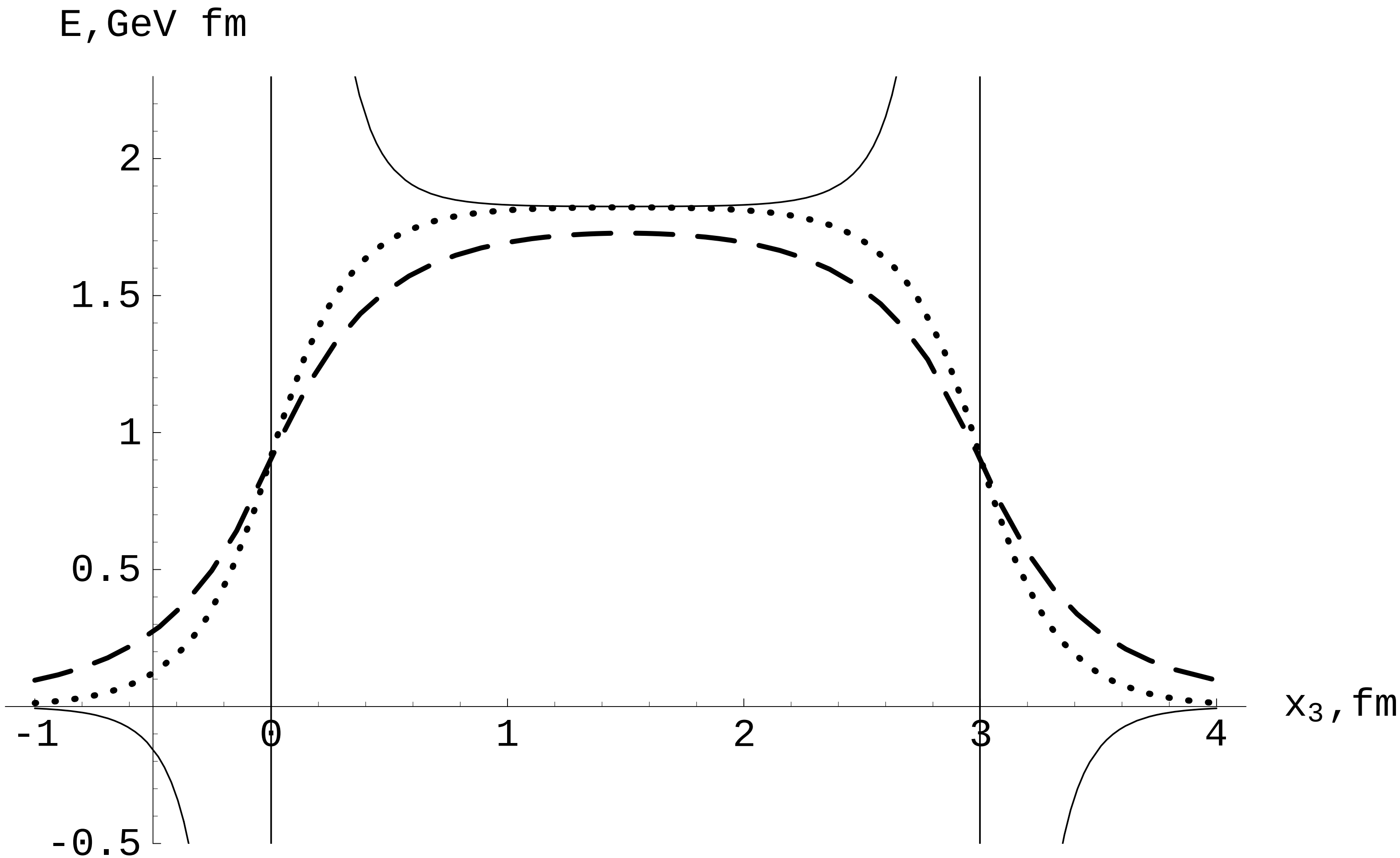

Figure 7: Distributions of the projections of the fields $\boldsymbol{\mathcal{E}}^D(0,0,x_3)$ (solid curve), $\boldsymbol{\mathcal{E}}^D(0,0,x_3) + \boldsymbol{\mathcal{E}}^{D_1,\mathrm{np}}(0,0,x_3)$ (dashed curve) and $\boldsymbol{\mathcal{E}}(0,0,x_3)$ (dotted curve) into the quark-antiquark axis at $Q\bar{Q}$-separation 3 fm.

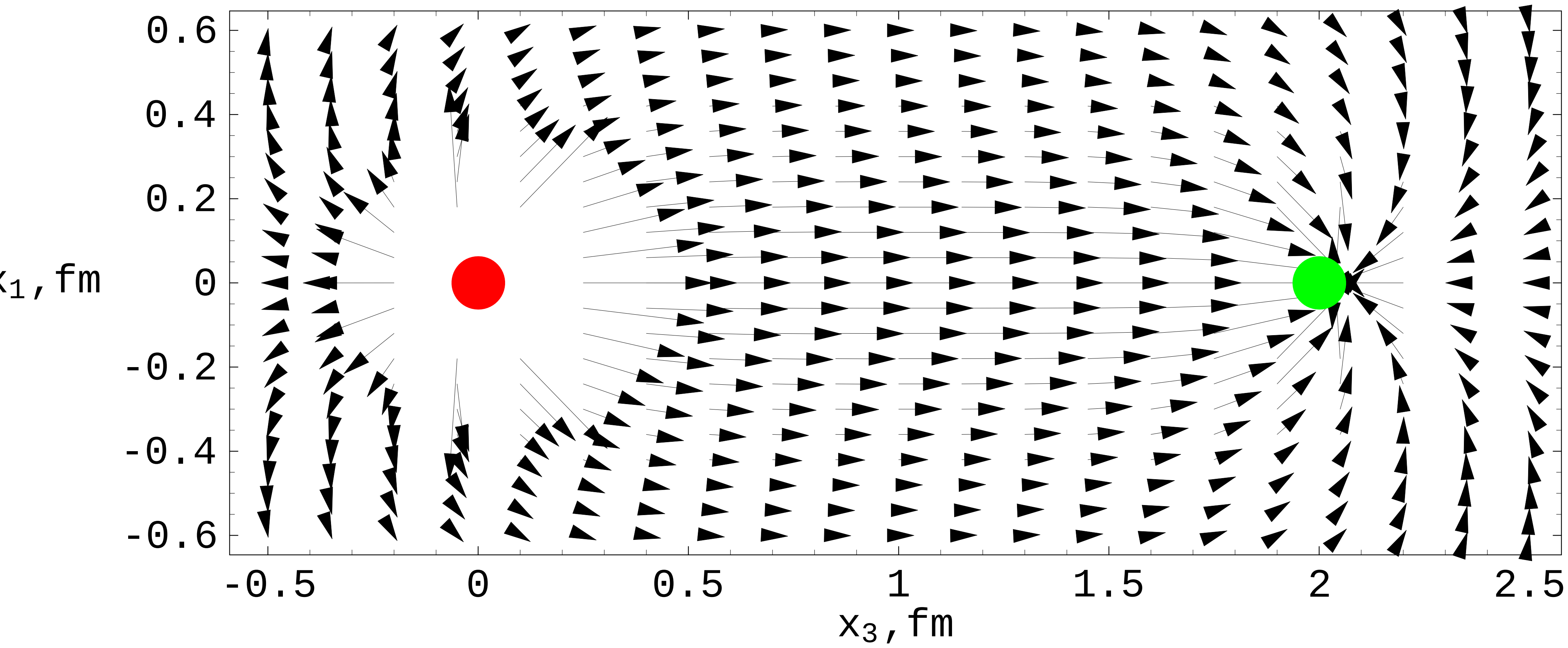

Figure 8: Vector distribution of the field $\boldsymbol{\mathcal{E}}(x_1, 0, x_3)$. Positions of quark and antiquark are marked by points.

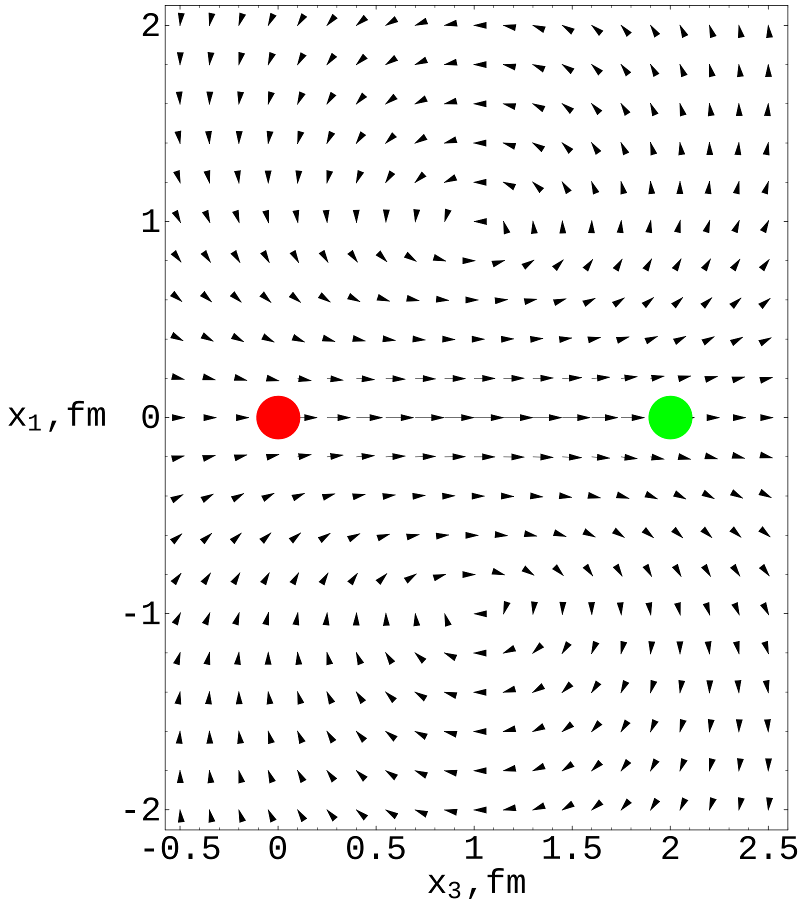

Figure 9: Vector distribution of the solenoid field $\boldsymbol{\mathcal{E}}^{D}(x_1, 0, x_3) + \boldsymbol{\mathcal{E}}^{D_1,\mathrm{np}}(x_1, 0, x_3)$. Positions of quark and antiquark are marked by points.

$$\boldsymbol{\mathcal{E}}^{D_1,\mathrm{np}} = \frac{\tilde{Q}(r)\,\mathbf{r}}{r^3} - \frac{\tilde{Q}(|\mathbf{r}-\mathbf{R}|)\,(\mathbf{r}-\mathbf{R})}{|\mathbf{r}-\mathbf{R}|^3}, \tag{144}$$

$$\tilde{Q}(r) = \frac{2\sigma\lambda^2}{\pi}\int_0^{r/\lambda} x^3\,K_1(x)dx, \tag{145}$$

$$Q(r) = C_F\alpha_s(r) - \tilde{Q}(r), \tag{146}$$

Condition of screening: perturbative fields are compensated by nonperturbative at large distances.

$$C_F\alpha_s = 3\sigma\lambda^2 \tag{147}$$

$$\varepsilon(r) = \frac{Q(r)}{C_F\alpha_s(r)}. \tag{148}$$

$$\varepsilon(r)|_{r\to\infty} = \frac{\sqrt{\pi}}{2}\left(\frac{r}{\lambda}\right)^{5/2}\exp\left(-\frac{r}{\lambda}\right) \tag{149}$$

Hadrons with three static sources

Green functions and W-loops

$$q^i(x,Y) \equiv q^j(x)\Phi^i_j(x,Y), \tag{150}$$

$$g_a(x,Y) \equiv g_b(x)\Phi_{ab}(x,Y), \tag{151}$$

$$B_Y(x,y,z,Y) = e_{ijk}q^i(x,Y)q^j(y,Y)q^k(z,Y), \tag{152}$$

$$G_Y^{(f)}(x,y,z,Y) = f^{abc}g_a(x,Y)g_b(y,Y)g_c(z,Y), \tag{153}$$

$$G_Y^{(d)}(x,y,z,Y) = d^{abc}g_a(x,Y)g_b(y,Y)g_c(z,Y), \tag{154}$$

$$G_\Delta(x,y,z) = G_i^j(x)\Phi_j^k(x,y)G_k^l(y)\Phi_l^m(y,z)G_m^n(z)\Phi_n^i(z,x). \tag{155}$$

$$\mathcal{G}_i(\bar{X},X) = \langle \Psi_i^+(\bar{X})\Psi_i(X)\rangle, \tag{156}$$

$$\langle \bar{q}_j(\bar{x}) q^i(x) \rangle \sim \Phi^i_j(\bar{x}, x),$$
$$\langle g_a(\bar{x}) g_b(x) \rangle \sim \Phi_{ab}(\bar{x}, x). \tag{157}$$

$$\mathcal{W}_B = \frac{1}{6} \langle \epsilon_{ijk} \epsilon^{i'j'k'} \Phi^i_{i'}(C_1) \Phi^j_{j'}(C_2) \Phi^k_{k'}(C_3) \rangle, \tag{158}$$
$$\mathcal{W}_G^{Y,f} = \frac{1}{24} \langle f^{abc} f^{a'b'c'} \Phi^{aa'}(C_1) \Phi^{bb'}(C_2) \Phi^{cc'}(C_3) \rangle, \tag{159}$$
$$\mathcal{W}_G^{Y,d} = \frac{3}{40} \langle d^{abc} d^{a'b'c'} \Phi^{aa'}(C_1) \Phi^{bb'}(C_2) \Phi^{cc'}(C_3) \rangle. \tag{160}$$

$$\mathcal{W}_G^{\triangle}(X, \bar{X}) = W(\bar{x}, \bar{y} | x, y) W(\bar{y}, \bar{z} | y, z) W(\bar{z}, \bar{x} | z, x). \tag{161}$$

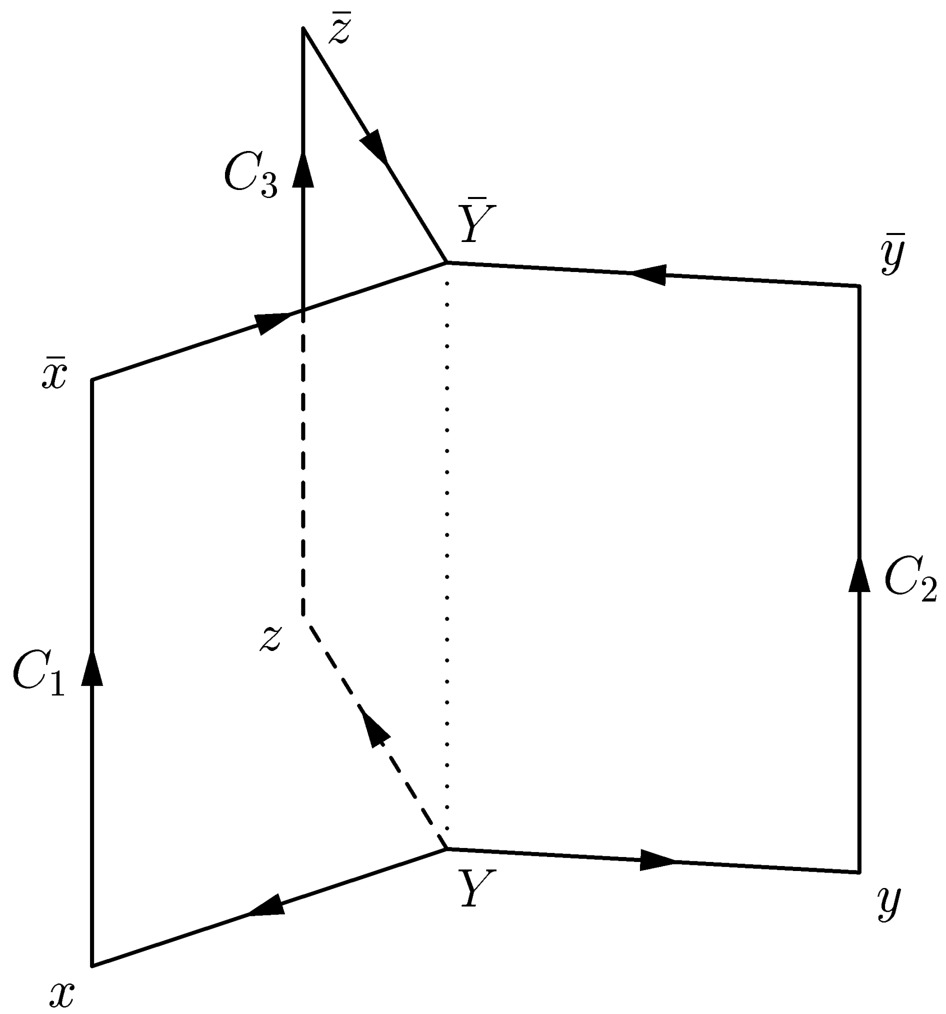

Figure 10: A W-loop of Y-type.

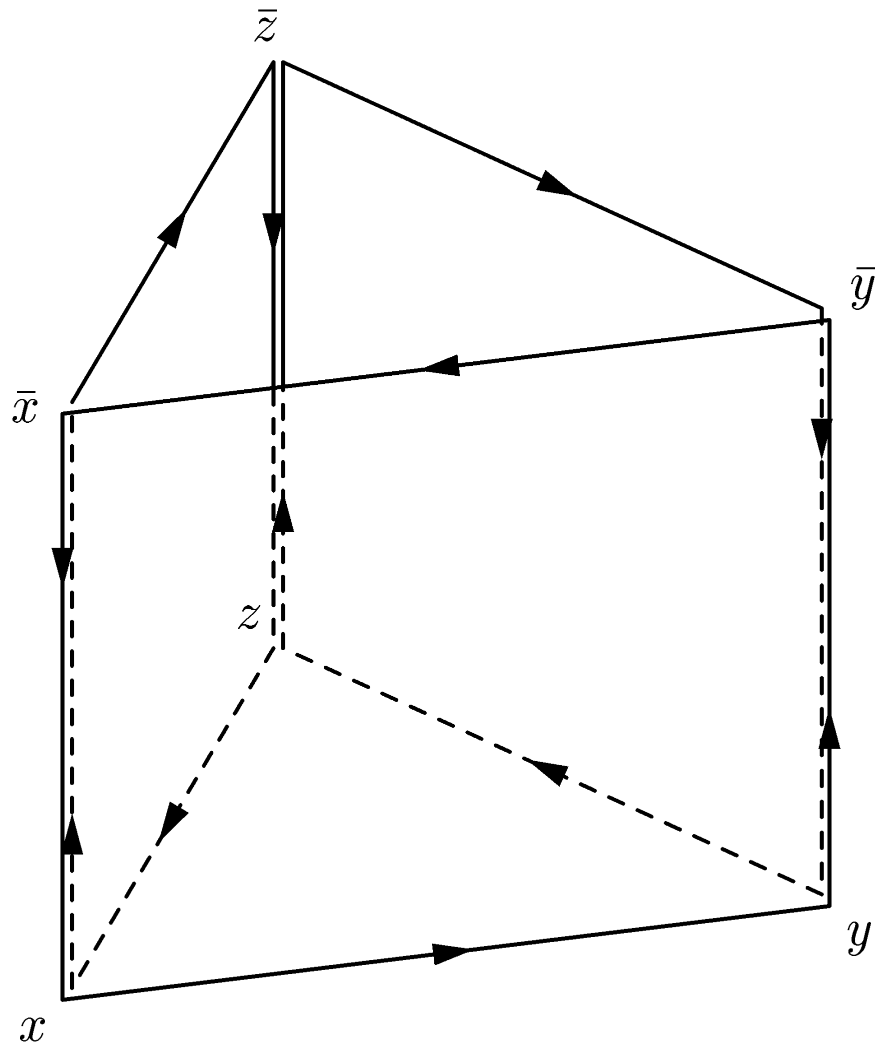

Figure 11: A W-loop of Δ-type.

Static potentials

$$V_B(R_1, R_2, R_3) = \left(\sum_{a=b} - \sum_{a<b} \right) n_i^{(a)} n_j^{(b)} \int_0^{R_a} \int_0^{R_b} dl\, dl' \int_0^{\infty} dt\, \mathcal{D}_{i4,j4}(z_{ab}), \tag{162}$$

$$V_B = V^c + V^d + V^{\mathrm{nd}}, \tag{163}$$

$$V^c = -\frac{C_F \alpha_s}{2} \sum_{i<j} \frac{1}{r_{ij}}, \tag{164}$$

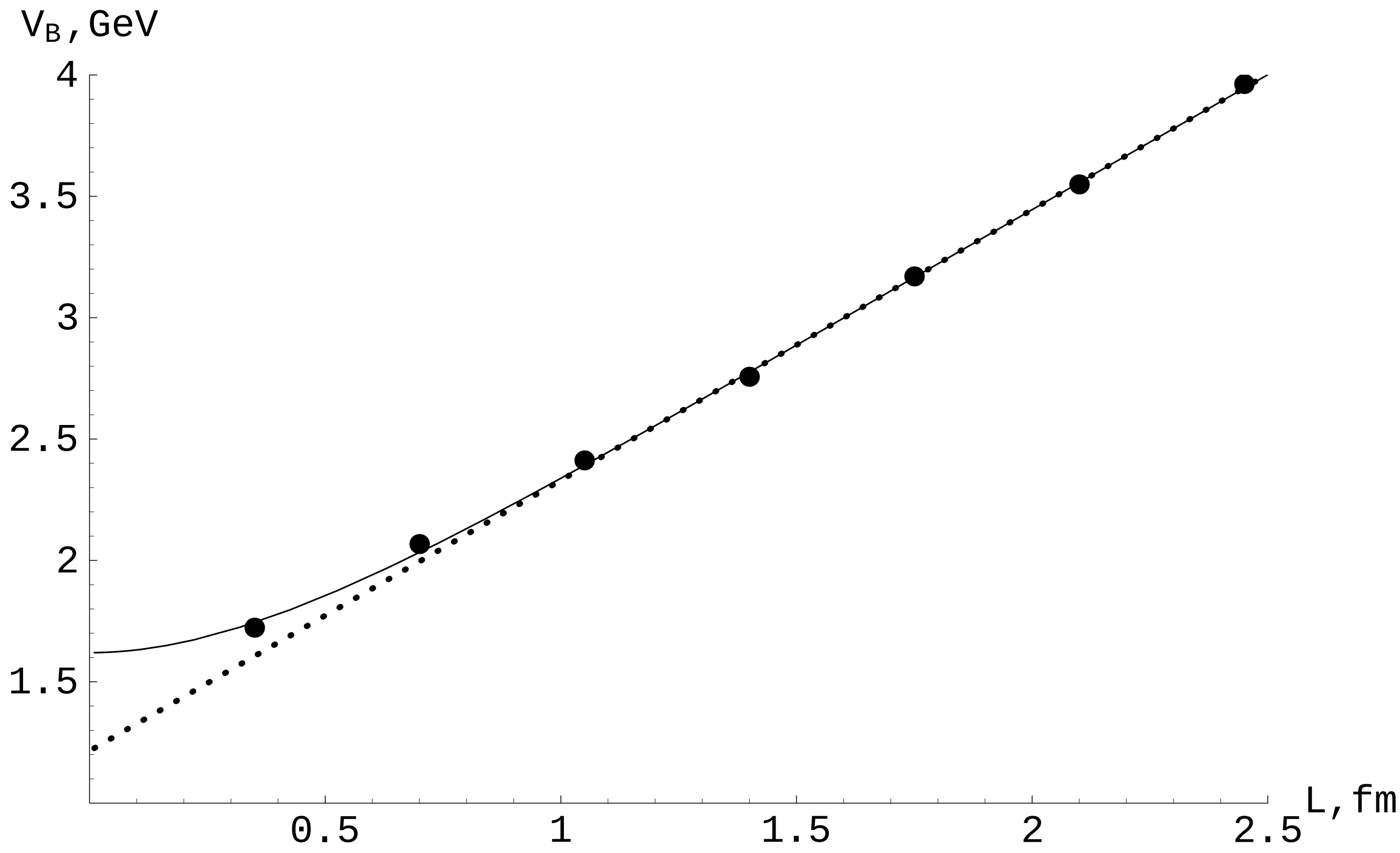

Figure 12: A potential in baryon (162) with the color-Coulomb part contracted (solid curve) in comparison with the lattice data points in dependence on the total length of the baryon string L. A value of the string tension is $\sigma = 0.22$ GeV^2. According to (**??**), the corresponding value of correlation length is $\lambda = 0.18$ fm.

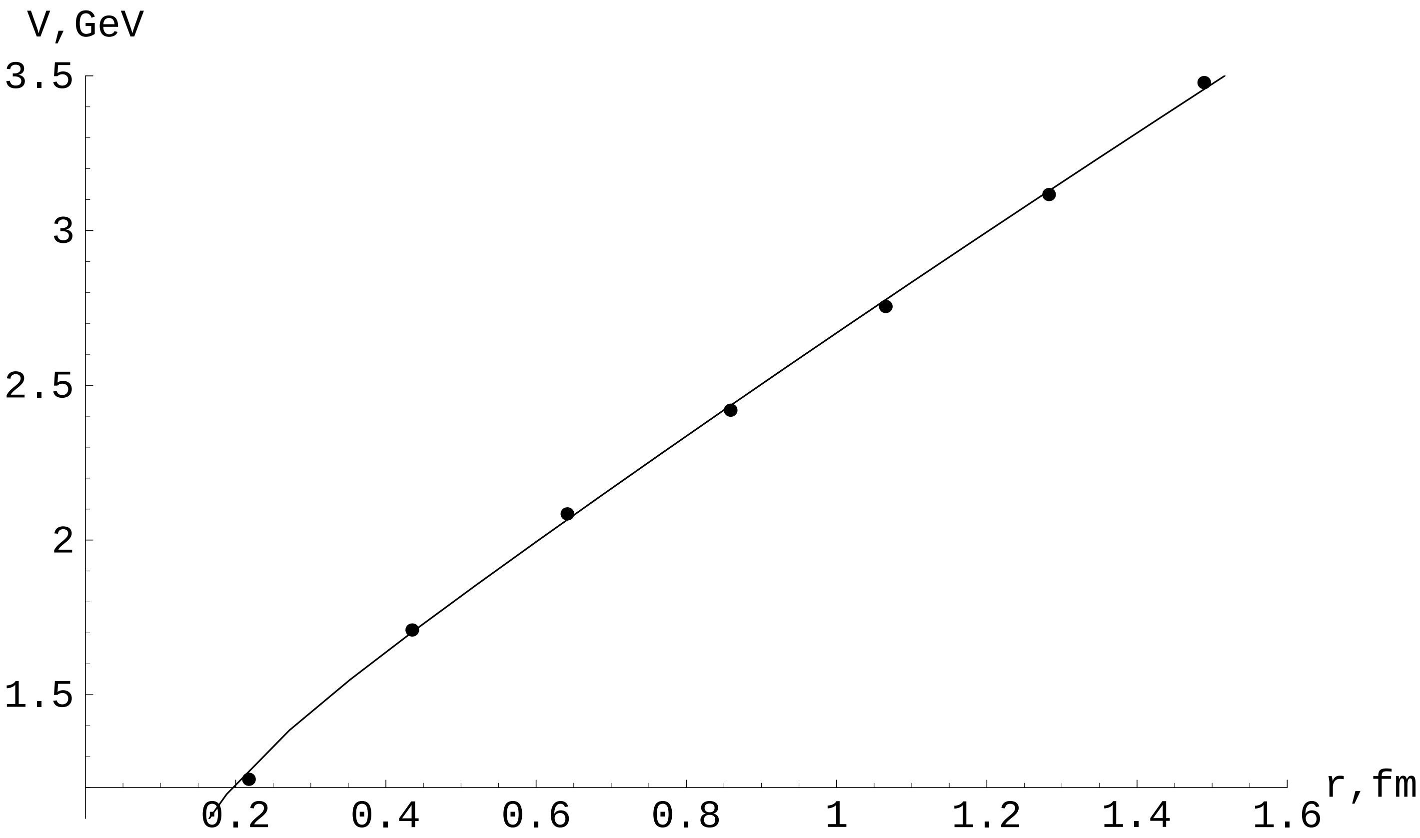

Figure 13: A dependence of the baryon potential in equilateral triangle on quark separation r (solid curve) in comparison with the lattice data [**?**] (points). A value of the string tension is $\sigma = 0.17$ GeV^2 ($\lambda = 0.21$ fm).

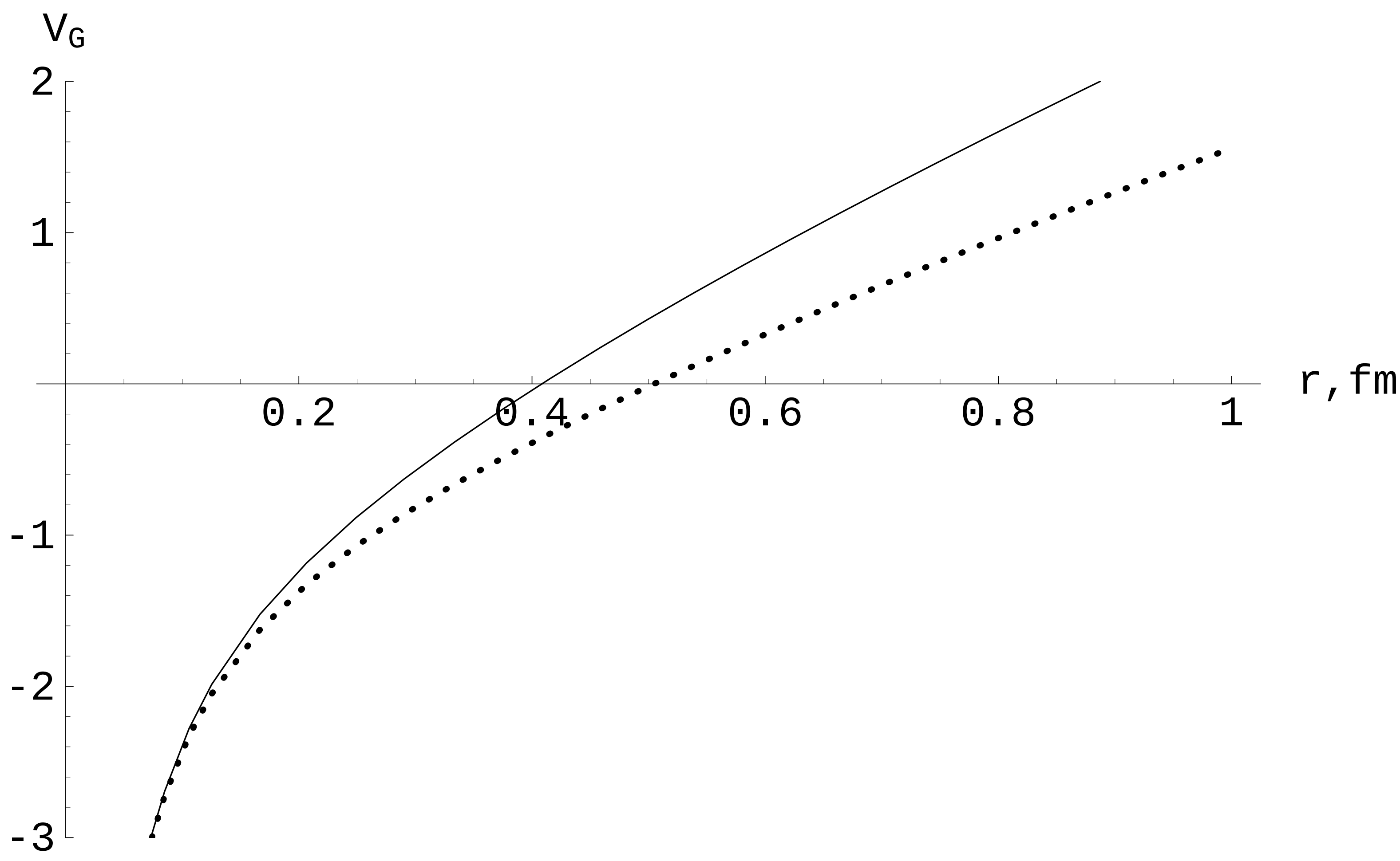

Figure 14: Potentials of three-gluon-glueballs V_G^Y (solid curve) and V_G^{Δ} (dotte curve) in equilateral triangle vs. the sources separation r.

$$V^d(R_1, R_2, R_3) = \sum_a V^D(R_a). \tag{165}$$

$$\frac{V_G^Y}{V_B} = \frac{C_8}{C_3}, \tag{166}$$

$$V_G^\Delta(r) = \frac{C_8}{C_3} V^c(r) + V^d(r) - 2V^{\mathrm{nd}}(r). \tag{167}$$

Fields distributions

$$(\boldsymbol{\mathcal{E}}^{(B)})^2 = \frac{2}{3}\,\Big((\boldsymbol{\mathcal{E}}^B_{(1)})^2 + (\boldsymbol{\mathcal{E}}^B_{(2)})^2 + (\boldsymbol{\mathcal{E}}^B_{(3)})^2\Big) \tag{168}$$

$$\boldsymbol{\mathcal{E}}^B_{(1)}(\mathbf{x}, \mathbf{R}^{(1)}, \mathbf{R}^{(2)}, \mathbf{R}^{(3)}) = \boldsymbol{\mathcal{E}}^M(\mathbf{x}, \mathbf{R}^{(1)}) - \frac{1}{2}\,\boldsymbol{\mathcal{E}}^M(\mathbf{x}, \mathbf{R}^{(2)}) - \frac{1}{2}\,\boldsymbol{\mathcal{E}}^M(\mathbf{x}, \mathbf{R}^{(3)}). \tag{169}$$

$$\boldsymbol{\mathcal{E}}^{(G)}_{\Delta}(\mathbf{x}, \mathbf{r}^{(1)}, \mathbf{r}^{(2)}, \mathbf{r}^{(3)}) = \sum_{i=1}^{3} \boldsymbol{\mathcal{E}}^M(\mathbf{x} - \mathbf{r}^{(i)}, \mathbf{r}^{(i+1)\mathrm{mod}3} - \mathbf{r}^{(i)}), \tag{170}$$

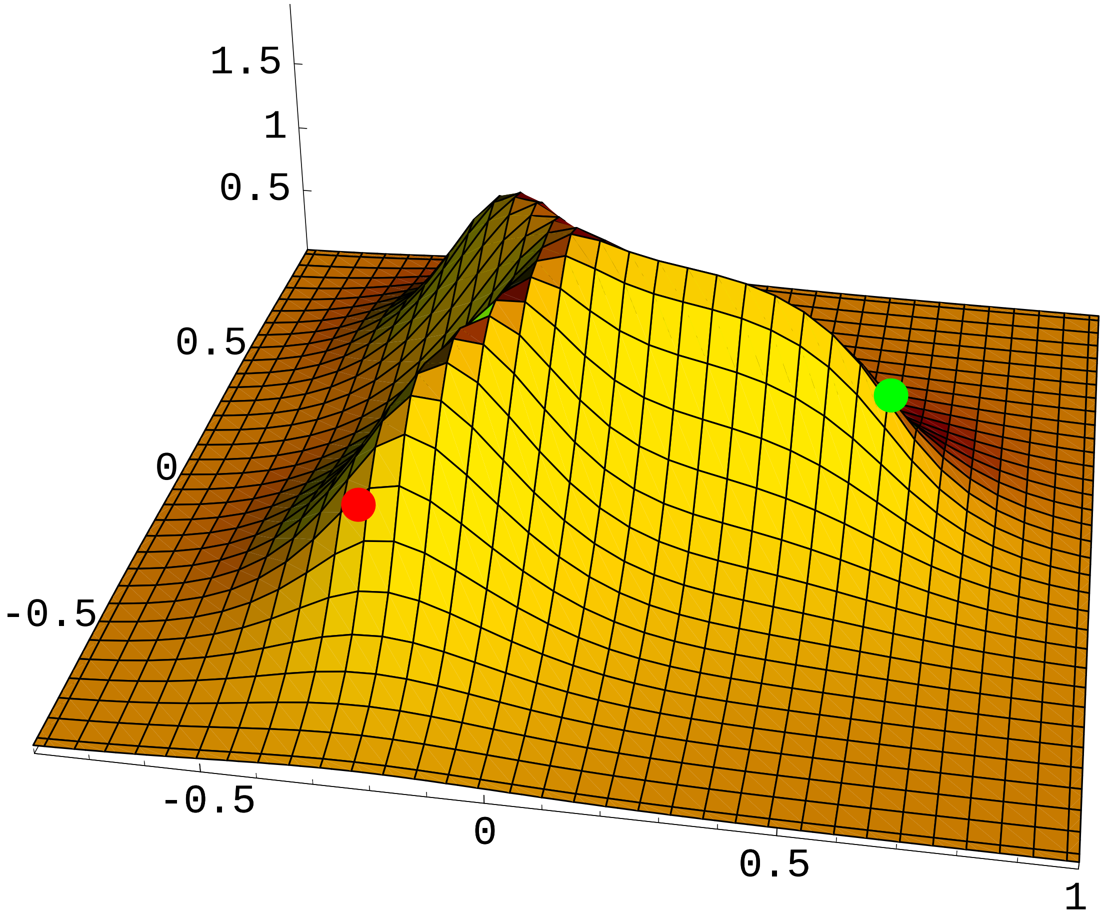

Figure 15: A distribution of the field $\boldsymbol{\mathcal{E}}^{(B)}$ (168), (169) in GeV/fm with the only correlator D contribution considered in the quark plane for equilateral triangle with the side 1 fm. Coordinates are given in fm, positions of quarks are marked by points.

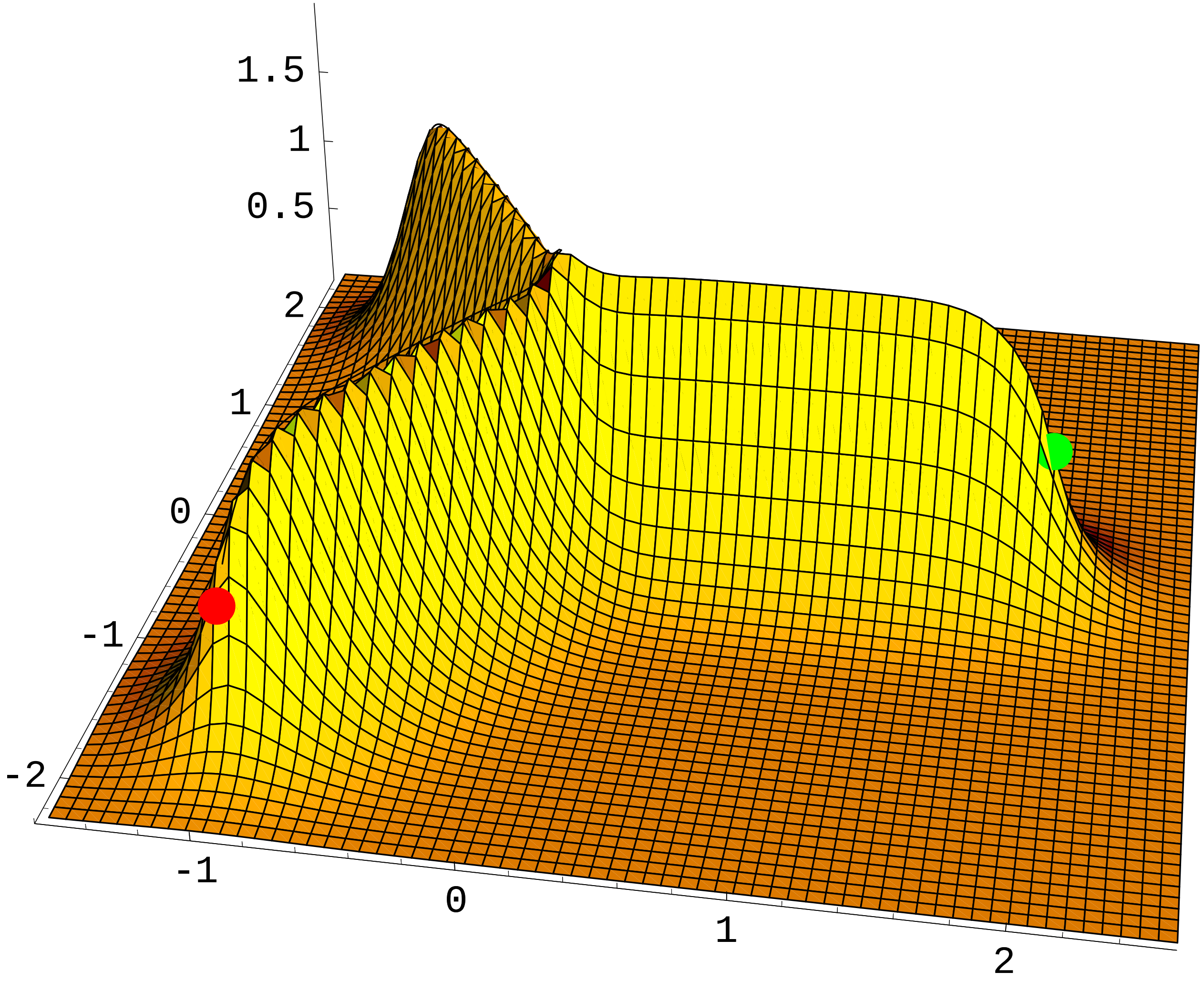

Figure 16: The same as in Fig. 15 but with the side of the equilateral triangle equal to 3.5 fm.

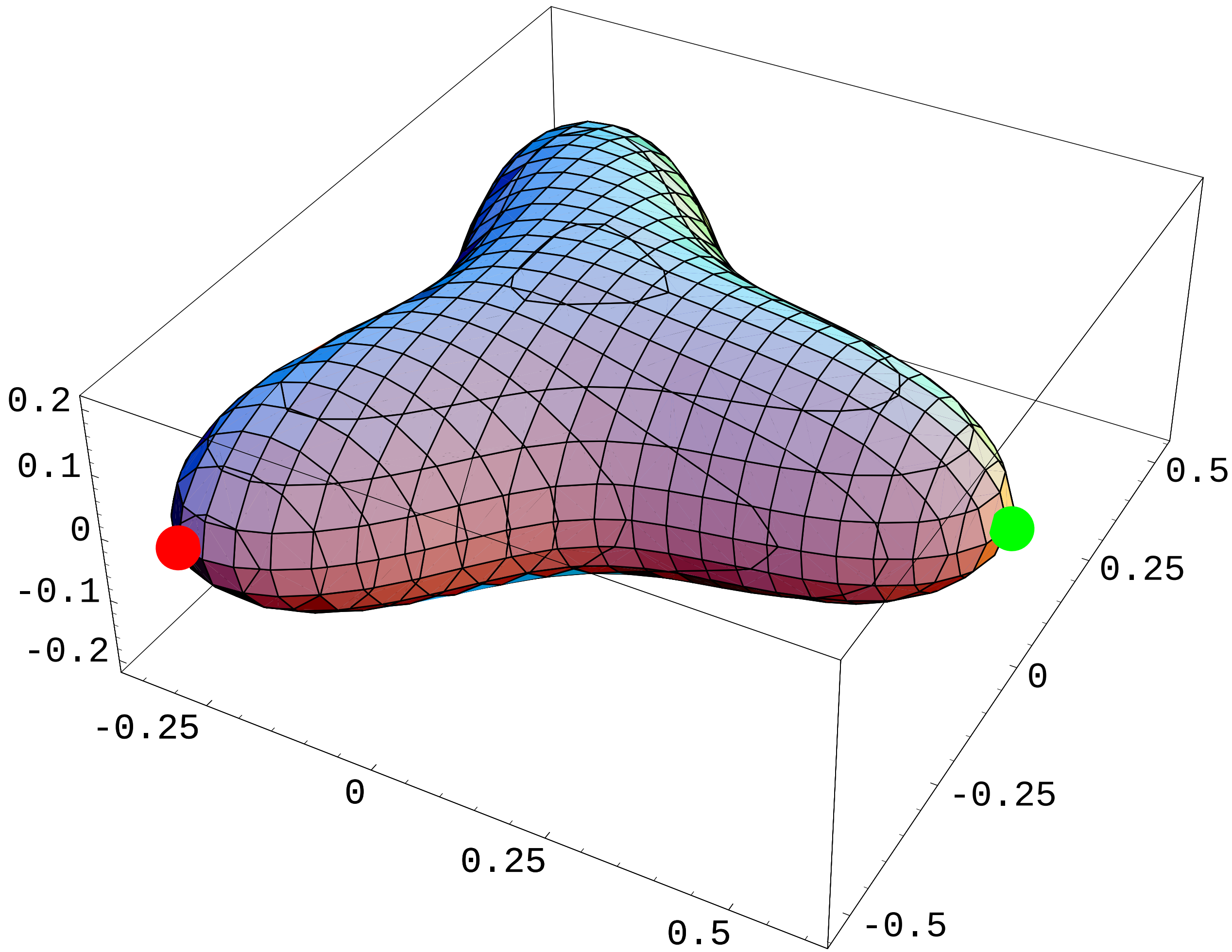

Figure 17: A surface $|\boldsymbol{\mathcal{E}}^{(B)}(\mathbf{x})| = \sigma$ at quark separations 1 fm. Coordinates are given in fm, positions of quarks are marked by points.

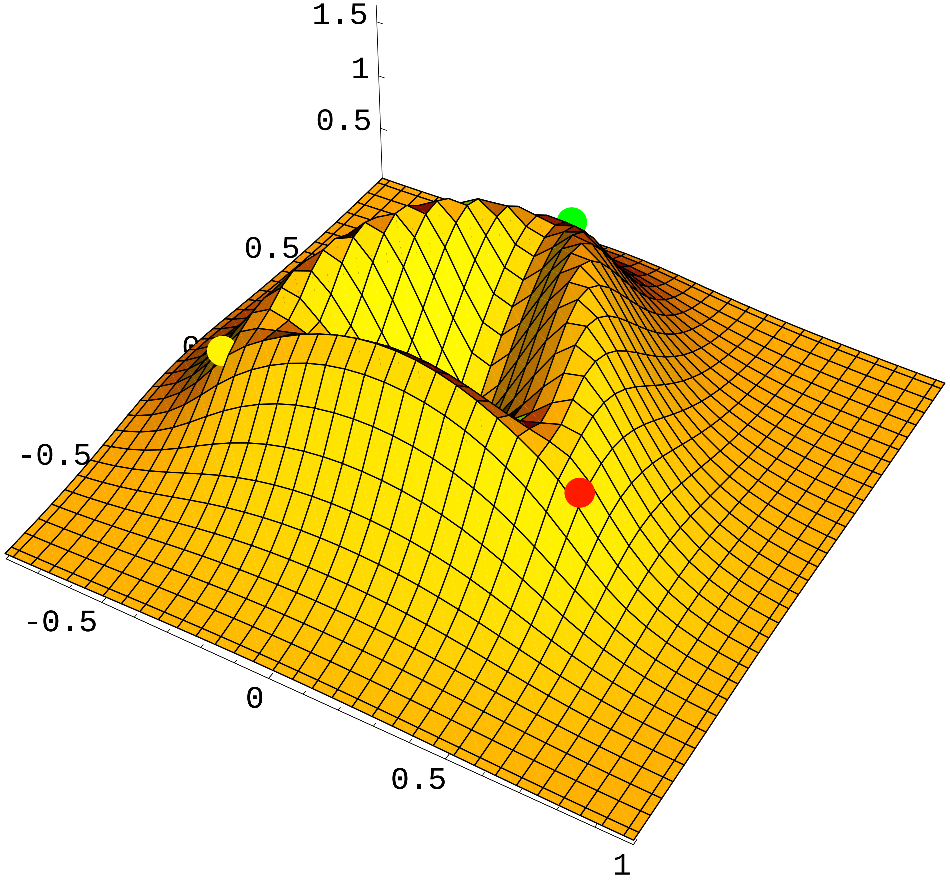

Figure 18: A distribution of the field $|\boldsymbol{\mathcal{E}}_{\Delta}^{(G)}(\mathbf{x})|$ (170) in GeV/fm of the triangular glueball in the plane of valence gluons with separations 1 fm. Coordinates are given in fm, positions of valence gluons are marked by points.

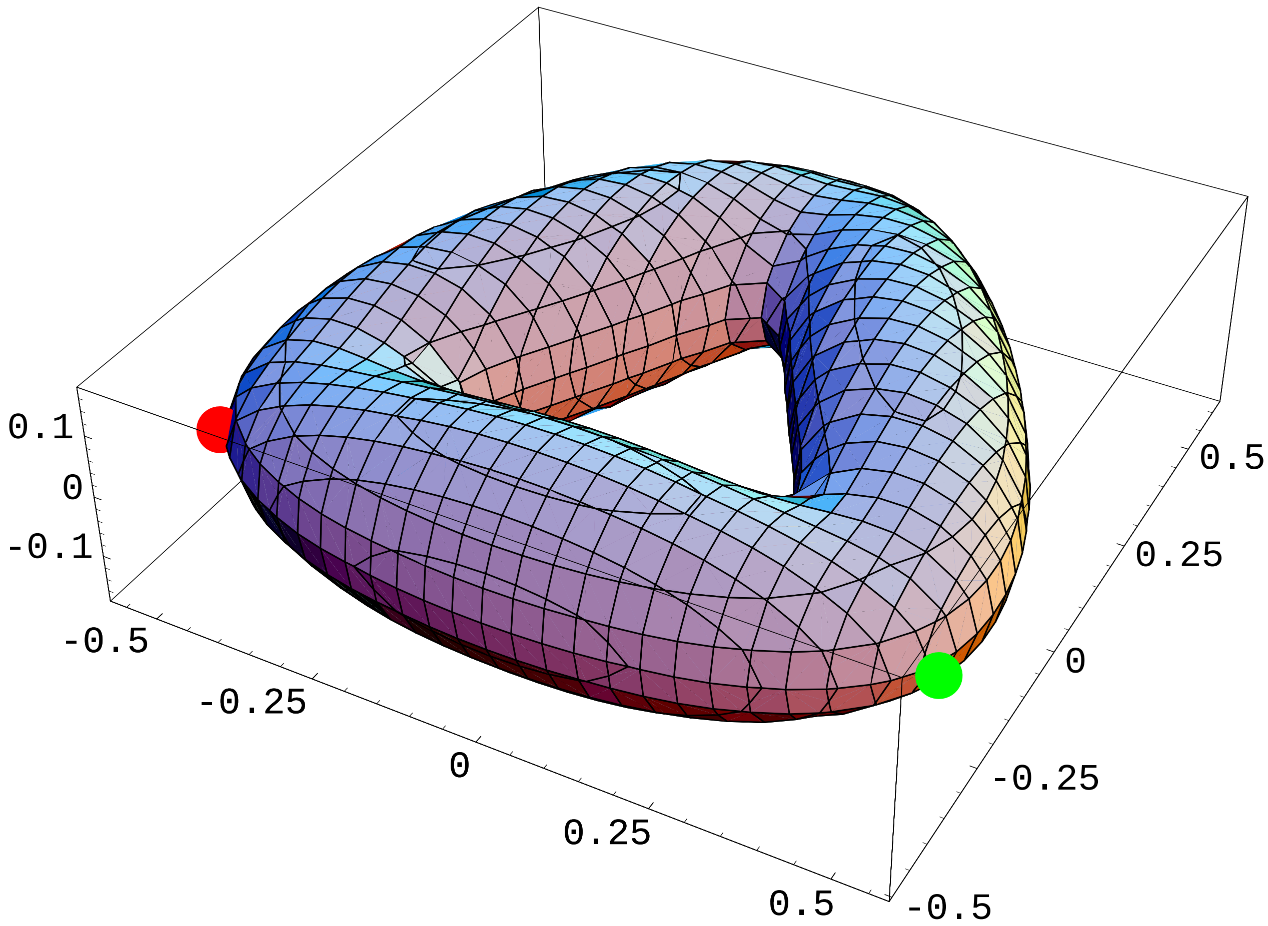

Figure 19: A surface $|\boldsymbol{\mathcal{E}}_{\Delta}^{(G)}(\mathbf{x})| = \sigma$ at valence gluons separations 1 fm. Coordinates are given in fm, positions of valence gluons are marked by points.

Conclusions

1. Field correlator Method is a powerful tool in QCD for treating nonperturbative as well as perturbative effects.

2. Bound states of quarks and gluons are computed in FCM with good accuracy and without fitting parameters. The input is minimal: current (pole) quark masses, string tension and α_s (or Λ_{QCD}).

3. Regge trajectories are obtained in good agreement with experiment for mesons and glueballs, and hybrids in agreement with lattice data.

4. Field distributions in mesons, glueballs and baryons display a clear image of the confining string.

5. One can interpret the string field and confinement as occurring due to effective magnetic currents confirming the dual Meissner picture of confinement. However this is unnecessary if one is working directly with Field Correlators.